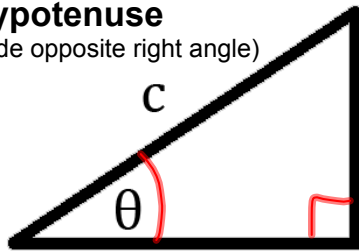


## 5.1 Trigonometric Functions of Acute Angles

An acute angle is an angle strictly between  $0^\circ$  and  $90^\circ$ .

A right triangle is a triangle with a  $90^\circ$  angle.

**hypotenuse**  
(side opposite right angle)



**opposite side**  
(side opposite angle of interest)

**adjacent side**  
(side adjacent to angle of interest)

$\theta$  theta

$\alpha$  alpha

$\beta$  beta

$\gamma$  gamma

The six basic trigonometric functions are ratios of sides of a right triangle.

sine  $\sin \theta = \frac{\text{length of side opposite } \theta}{\text{length of hypotenuse}} = \frac{\text{opp}}{\text{hyp}}$

cosine  $\cos \theta = \frac{\text{length of side adjacent to } \theta}{\text{length of hypotenuse}} = \frac{\text{adj}}{\text{hyp}}$

tangent  $\tan \theta = \frac{\text{length of side opposite } \theta}{\text{length of side adjacent to } \theta} = \frac{\text{opp}}{\text{adj}}$

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secant  $\sec \theta = \frac{\text{hyp}}{\text{adj}} = \frac{1}{\cos \theta}$

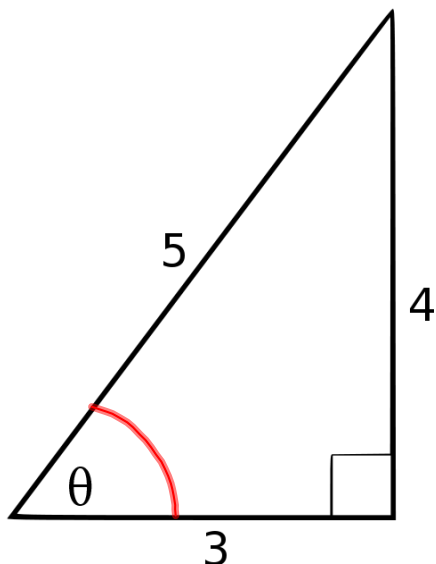
cosecant  $\csc \theta = \frac{\text{hyp}}{\text{opp}} = \frac{1}{\sin \theta}$

cotangent  $\cot \theta = \frac{\text{adj}}{\text{opp}} = \frac{1}{\tan \theta}$

$$\cos \theta = \frac{1}{\sec \theta}$$

$$\sin \theta = \frac{1}{\csc \theta}$$

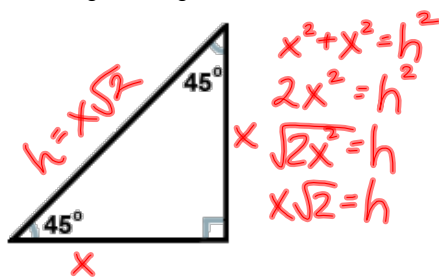
$$\tan \theta = \frac{1}{\cot \theta}$$



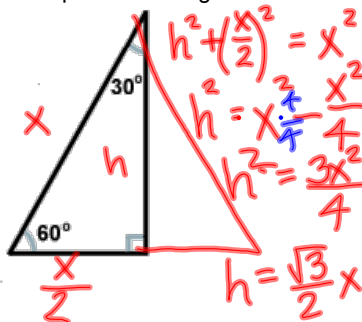
$$\begin{aligned} \sin \theta &= \frac{4}{5} \\ \cos \theta &= \frac{3}{5} \\ \tan \theta &= \frac{4}{3} \\ \sec \theta &= \frac{5}{3} \\ \csc \theta &= \frac{5}{4} \\ \cot \theta &= \frac{3}{4} \end{aligned}$$

Two special right triangles:

Isosceles Right Triangle aka 45-45-90



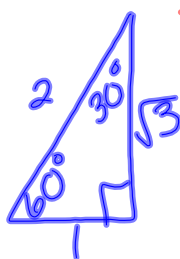
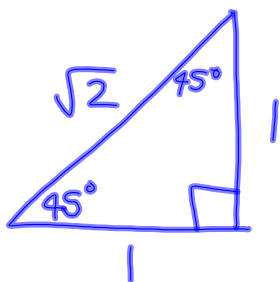
Half of an equilateral triangle aka 30-60-90



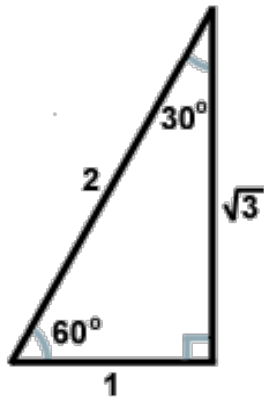
Pythagorean Theorem:

$$a^2 + b^2 = c^2$$

, a, b - legs of a right  $\Delta$   
 & c - hypotenuse



$$\begin{aligned} 30^\circ &< 60^\circ < 90^\circ \\ 1 &< 3 < 4 \\ \sqrt{1} &< \sqrt{3} < \sqrt{4} \\ 1 &< \sqrt{3} < 2 \end{aligned}$$



$$\begin{array}{l}
 \sin 30^\circ = \frac{1}{2} \\
 \cos 30^\circ = \frac{\sqrt{3}}{2} \\
 \tan 30^\circ = \frac{1}{\sqrt{3}} \\
 \sec 30^\circ = \frac{2}{\sqrt{3}} \\
 \csc 30^\circ = 2 \\
 \cot 30^\circ = \sqrt{3}
 \end{array}
 \quad
 \begin{array}{l}
 \sin 60^\circ = \frac{\sqrt{3}}{2} \\
 \cos 60^\circ = \frac{1}{2} \\
 \tan 60^\circ = \sqrt{3} \\
 \sec 60^\circ = 2 \\
 \csc 60^\circ = \frac{2}{\sqrt{3}} \\
 \cot 60^\circ = \frac{1}{\sqrt{3}}
 \end{array}$$

### Cofunctions

The function of an angle is equal to the cofunction of its complement.

The complement of an angle  $\theta$  is  $90^\circ - \theta$ .

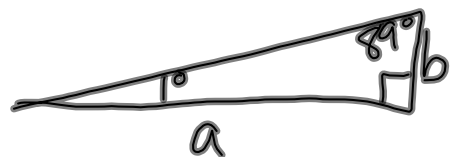
$$\sin(90^\circ - \theta) = \cos \theta; \quad \tan \theta = \cot(90^\circ - \theta)$$

Examples:

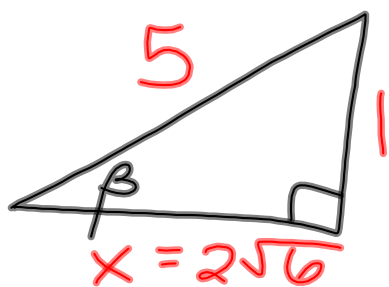
$$\csc 20^\circ = \sec 70^\circ$$

$$\tan 89^\circ = \cot 1^\circ$$

$$\cos 7^\circ = \sin 83^\circ$$



Given that  $\csc \beta = 5$ , find the other trigonometric function values of  $\beta$ .



$$x^2 + 1^2 = 5^2$$

$$x^2 = 25 - 1$$

$$x = \sqrt{24} = 2\sqrt{6}$$

$$\sin \beta = \frac{1}{5}$$

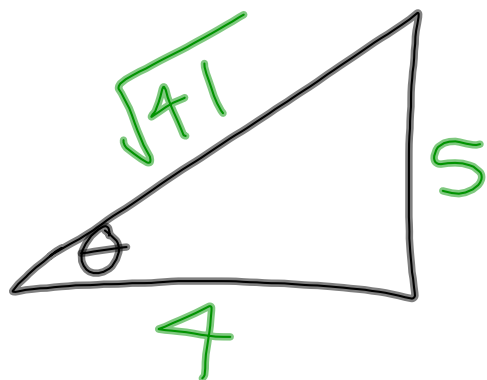
$$\cos \beta = \frac{2\sqrt{6}}{5}$$

$$\tan \beta = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$$

$$\cot \beta = 2\sqrt{6}$$

$$\sec \beta = \frac{5}{2\sqrt{6}}$$

Given that  $\cot \theta = \frac{4}{5}$ , find the other trigonometric function values of  $\theta$ .



$$\sin \theta = \frac{5}{\sqrt{41}}$$

$$\cos \theta = \frac{4}{\sqrt{41}}$$

$$\tan \theta = \frac{5}{4}$$

$$\sec \theta = \frac{\sqrt{41}}{4}$$

$$\csc \theta = \frac{\sqrt{41}}{5}$$

**Homework:**

5.1 #1-15 odd; ~~16~~-28 all  
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