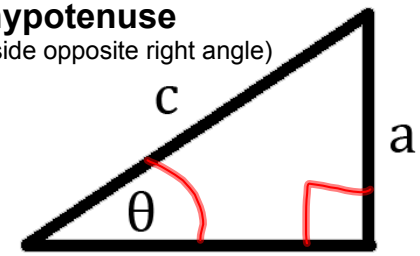
5.1 Trigonometric Functions of Acute Angles

An acute angle is an angle strictly between 0° & 90°.

A right triangle is a triangle with a 90° angle.



hypotenuse
(side opposite right angle)

opposite side
(side opposite angle of interest)

adjacent side
(side adjacent to angle of interest)

θ theta

α alpha

β beta

γ gamma

The six basic trigonometric functions are ratios of sides of a right triangle.

sine $\sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse} = \frac{opp}{hyp}$

cosine $\cos\theta = \frac{length\ of\ side\ adjacent\ to\ \theta}{length\ of\ hypotenuse} = \frac{adj}{hyp}$

tangent $\tan\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ to\ \theta} = \frac{opp}{adj}$

SohCahToa

secant $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

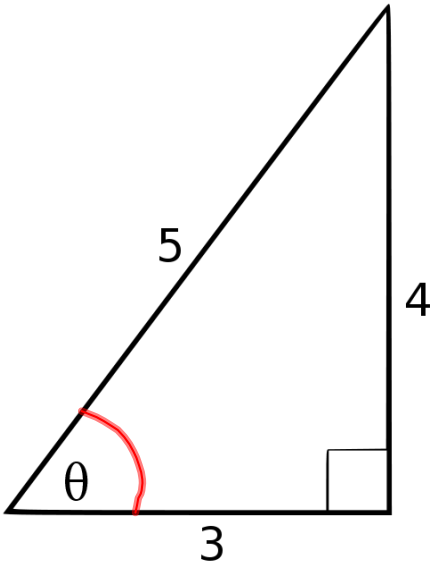
cosecant $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

cotangent $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta}$

$\cos\theta = \frac{1}{\sec\theta}$

$\sin\theta = \frac{1}{\csc\theta}$

$\tan\theta = \frac{1}{\cot\theta}$



$\sin\theta = 4/5$

$\cos\theta = 3/5$

$\tan\theta = 4/3$

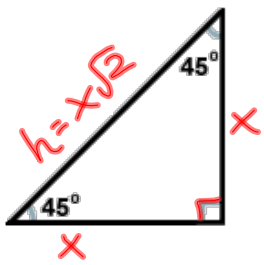
$\sec\theta = 5/3$

$\csc\theta = 5/4$

$\cot\theta = 3/4$

Two special right triangles:

Isosceles Right Triangle aka 45-45-90



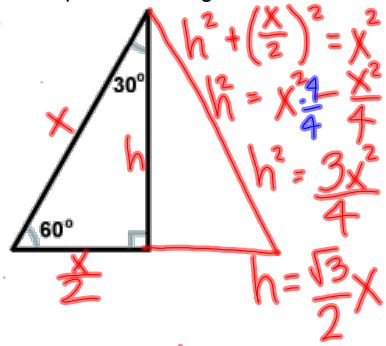
$x^2+x^2=h^2$

$2x^2=h^2$

$\sqrt{2x^2}=h$

$x\sqrt{2}=h$

Half of an equilateral triangle aka 30-60-90



$h^2+\left(\frac{x}{2}\right)^2=x^2$

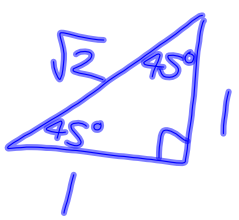
$h^2=x^2-\frac{x^2}{4}$

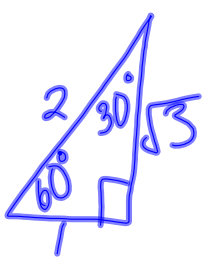
$h^2=\frac{3x^2}{4}$

$h=\frac{\sqrt{3}}{2}x$

Pythagorean Theorem:

$a^2+b^2=c^2$, a,b - legs of right △, c - hypotenuse



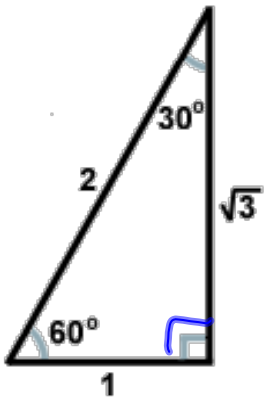


$30^\circ < 60^\circ < 90^\circ$

$1 < 3 < 4$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

$1 < \sqrt{3} < 2$



$\sin 30° = \frac{1}{2}$ $\quad$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\quad$ $\tan 60° = \sqrt{3}$

$\sec 30° = \frac{2}{\sqrt{3}}$ $\quad$ $\sec 60° = 2$

$\csc 30° = 2$ $\quad$ $\csc 60° = \frac{2}{\sqrt{3}}$

$\cot 30° = \sqrt{3}$ $\quad$ $\cot 60° = \frac{1}{\sqrt{3}}$

Cofunctions

The function of an angle is equal to the cofunction of its complement.

The complement of an angle θ is $90° - \theta$.

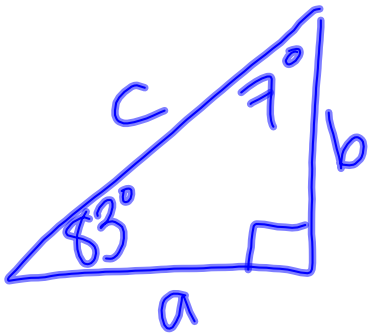
$\sin(90° - \theta) = \cos\theta$; $\cot\theta = \tan(90° - \theta)$

Examples:

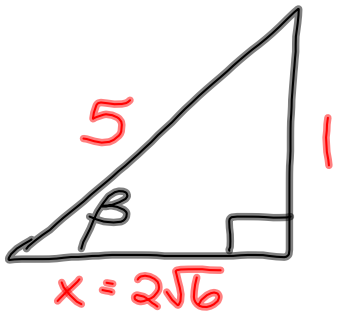
$\cos(20°) = \sin 70°$

$\csc(89°) = \sec 1°$

$\tan(7°) = \cot 83°$



Given that $\csc\beta = 5$, find the other trigonometric function values of β.



$x^2 + 1^2 = 5^2$

$x^2 = 25 - 1$

$x = \sqrt{24} = 2\sqrt{6}$

$\sin\beta = \frac{1}{5}$

$\cos\beta = 2\sqrt{6}/5$

$\tan\beta = \frac{1}{2\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{12}$

$\sec\beta = \frac{5}{2\sqrt{6}}$

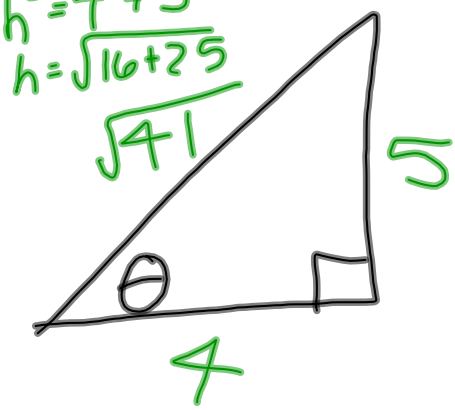
$\cot\beta = 2\sqrt{6}$

Given that $\cot\theta = \frac{4}{5}$, find the other trigonometric function values of θ.

$h^2 = 4^2 + 5^2$

$h = \sqrt{16+25}$

$\sqrt{41}$



$\sin\theta = 5/\sqrt{41}$

$\cos\theta = 4/\sqrt{41}$

$\tan\theta = 5/4$

$\sec\theta = \sqrt{41}/4$

$\csc\theta = \sqrt{41}/5$

Homework:

5.1 #1-15 odd; 16-28 all

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