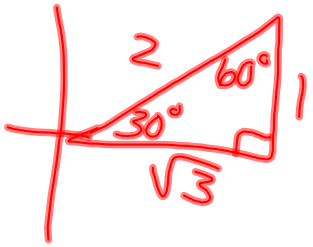
Review:

A reference angle for an angle whose inial side is on the posive x-axis and terminal side may lie in any of the four quadrants is ~~the acute angle~~ between the terminal side of the given angle and the X-axis.

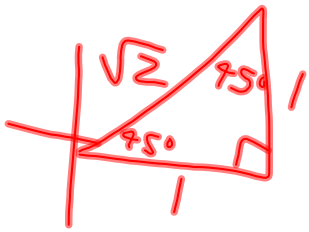
Evaluate the following trigonometric expressions. Give exact answers. You do not have to raonalize. Draw a picture if this helps you.

$\sin 45°$ $\frac{1}{\sqrt{2}}$



$\tan 60°$ $\sqrt{3}$

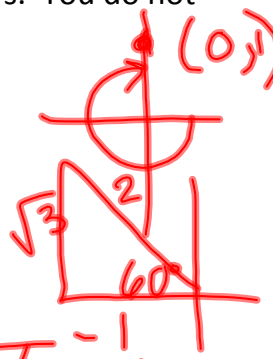
$\sec 45°$ $\sqrt{2}$

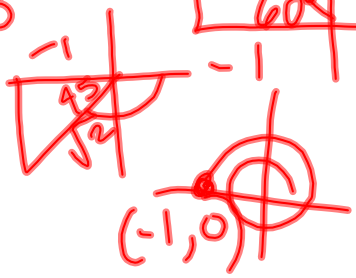


$\csc 30°$ 2

$\sec(-270°)$ undefined

$\cot(120°)$ $-\frac{1}{\sqrt{3}}$

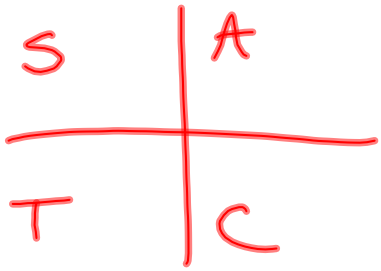


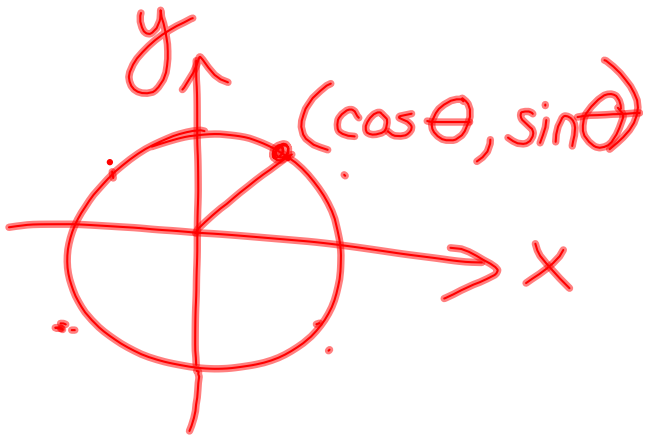


$\csc(-135°)$ $-\sqrt{2}$

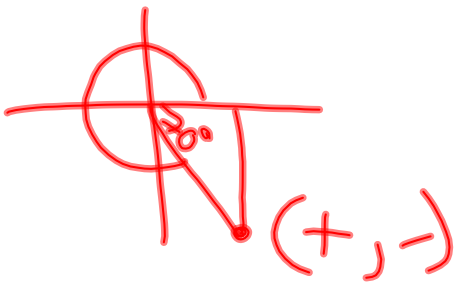
$\tan(540°)$ 0

Homework questions?





76.290°



cos, sec +

sin, csc, tan, cot −

5.4 Radians

The circumference of a circle of radius r is given by the equation:

$C = 2\pi r$

Therefore, the unit circle, which has radius 1, has circumference:

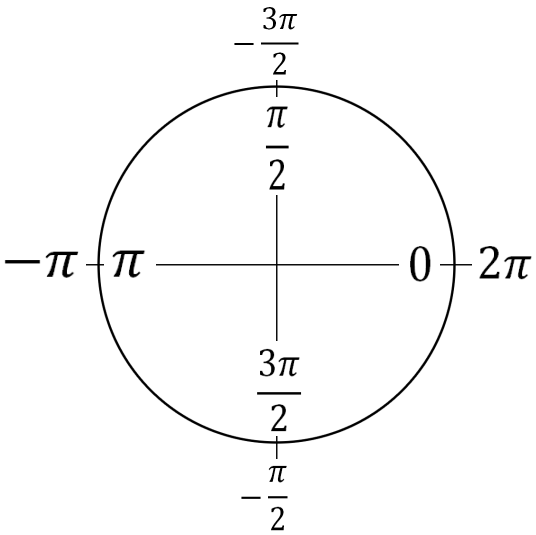
2π

The irrational number pi is approximately: $\pi \approx 3.14$

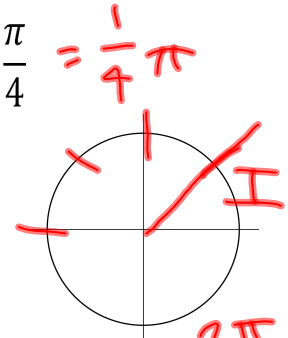
Therefore $2\pi \approx 6.28$

$4\pi \approx 12.56$

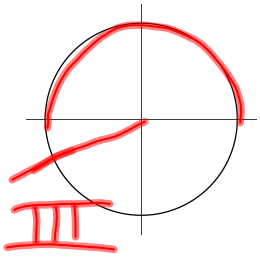
If we think about these numbers as corresponding to arc lengths around the unit circle, in which quadrant (or on which axis) do we end up?



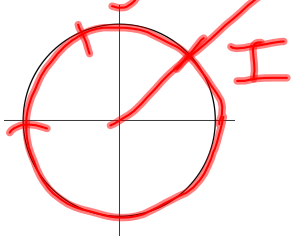
$\frac{\pi}{4} = \frac{1}{4}\pi$

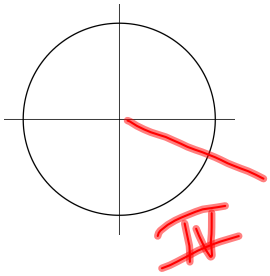


$\frac{7\pi}{6} = \frac{6\pi}{6} + \frac{\pi}{6}$



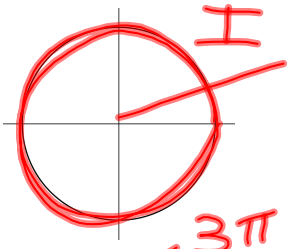
$\frac{7\pi}{3} = \frac{6\pi}{3} + \frac{\pi}{3}$ 6

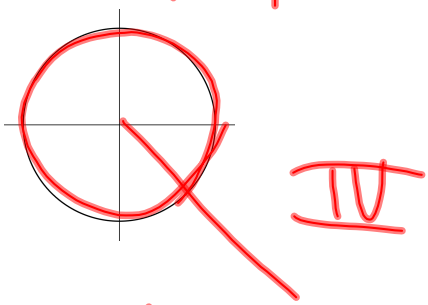




13

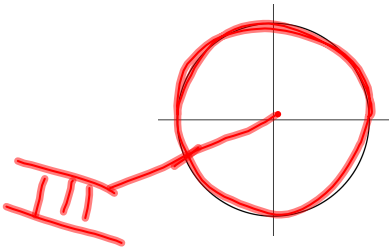
$-\frac{9\pi}{4} = -\frac{8\pi}{4} - \frac{\pi}{4} = -2\pi - \frac{\pi}{4}$



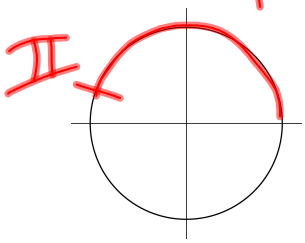


$= 3\pi$

$\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6}$



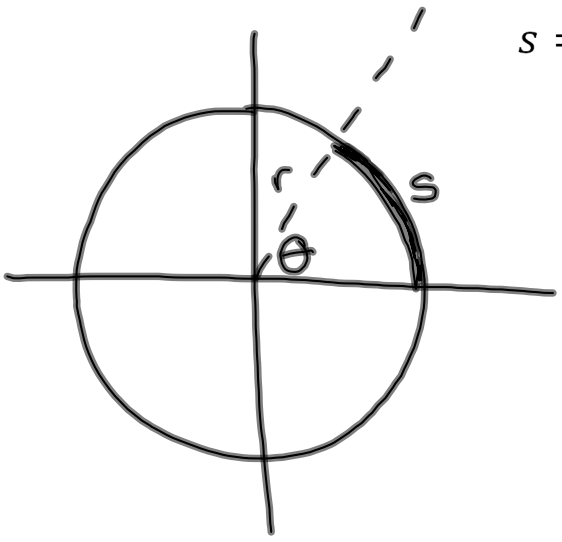
$\frac{6\pi}{7} = \frac{6}{7}\pi$



What is a radian?

r = radius length

s = arc length



When $s = r$, we say that the corresponding angle θ which is subtended by arc s has measure 1 radian.

$1 \text{ radian} \approx 57.3°$

$\pi = 180°$

$2\pi = 360°$

Note that θ is independent of the radius length and any unit of measurement. Therefore radians have no associated units, and any angle measure without a degree symbol is assumed to be in radians.

Converting between radians and degrees

$$\pi = 180^\circ \quad \therefore \quad \frac{\pi}{180^\circ} = 1 = \frac{180^\circ}{\pi}$$

Convert 225° to radians.

$225 \cdot \frac{\pi}{180} = \frac{5\pi}{4}$

Convert $\frac{5\pi}{6}$ to degrees.

$\frac{5\pi}{6} \cdot \frac{180^\circ}{\pi} = 150^\circ$

Convert 120° to radians.

$120^\circ \cdot \frac{\pi}{180^\circ} = \frac{2\pi}{3}$

Convert $\frac{7\pi}{4}$ to degrees.

$\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = 315^\circ$

Two angles in radians are:

complementary if they sum to $\frac{\pi}{2}$. (90°)

supplementary if they sum to π. (180°)

coterminal if they differ by integer multiples of 2π. (360°)

Find the complement and supplement of $\frac{5\pi}{12}$.

comp: $\frac{\pi}{2} - \frac{5\pi}{12} = \frac{6\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{\pi}{12}}$ supp: $\pi - \frac{5\pi}{12} = \frac{12\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{7\pi}{12}}$

Find one positive and one negative angle coterminal with $-\frac{3\pi}{4}$.

$-\frac{3\pi}{4} + \frac{2\pi}{1}\cdot\frac{4}{4} = -\frac{3\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{5\pi}{4}}$; $-\frac{3\pi}{4} - \frac{8\pi}{4} = \boxed{-\frac{11\pi}{4}}$

Common angles:
(memorize!)

$\frac{\pi}{6} = 30°$

$\frac{\pi}{4} = 45°$

$\frac{\pi}{3} = 60°$

Note:

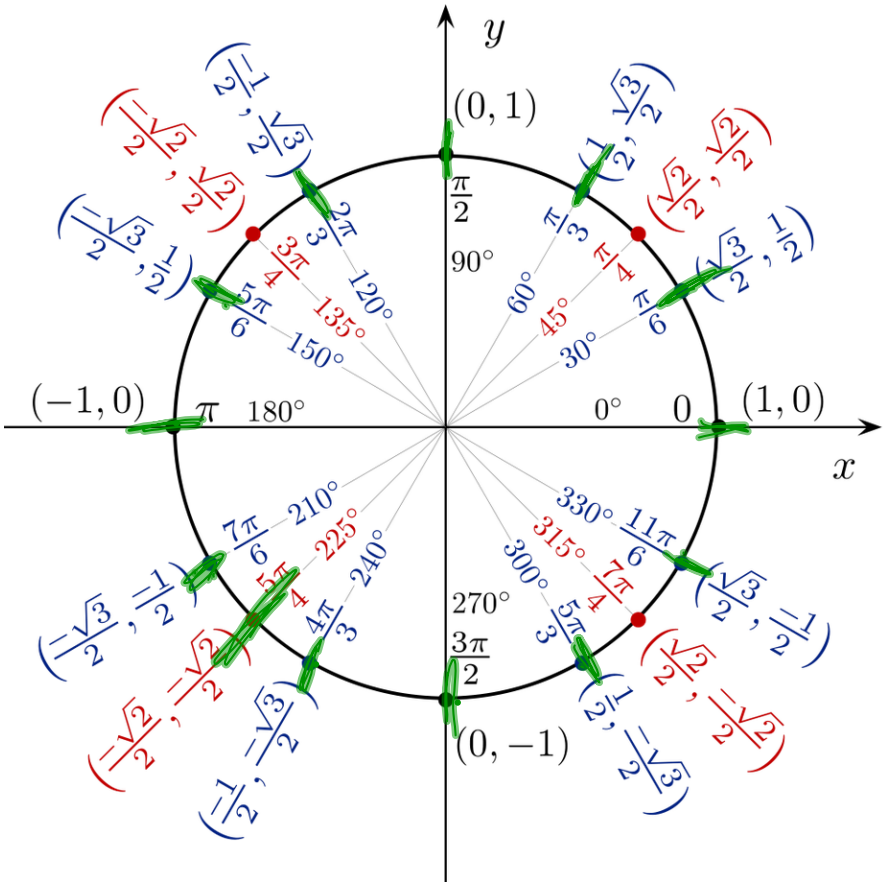
$\frac{k\pi}{6} \rightarrow 30°\ ref.\angle$

$\frac{k\pi}{4} \rightarrow 45°\ ref.\angle$

$\frac{k\pi}{3} \rightarrow 60°\ ref.\angle$

$\frac{k\pi}{2} \rightarrow 90°\ or\ 270°$

$k\pi \rightarrow 0°\ for\ k\ even;$
$180°\ for\ k\ odd$



Homework:

5.4

#1-7 odd - determining quadrant/location of angles in radians

#9-19 odd - compliment/supplement/coterminal angles

#21,23,27,31,45,47,53 - convert between radians and degrees

Next time:

- determine trigonometric function value of angles given in radians
- arc length/linear speed/angular speed problems