

Review:

A reference angle for an angle whose initial side is on the positive x-axis and terminal side may lie in any of the four quadrants is the acute angle between the terminal side of the given angle and the x-axis.

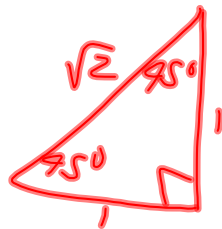
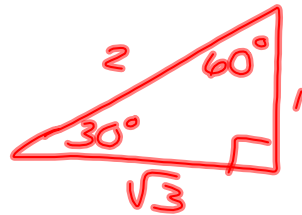
Evaluate the following trigonometric expressions. Give exact answers. You do not have to rationalize. Draw a picture if this helps you.

$\sin 45^\circ = \frac{1}{\sqrt{2}}$

$\tan 60^\circ = \sqrt{3}$

$\sec 45^\circ = \sqrt{2}$

$\csc 30^\circ = 2$

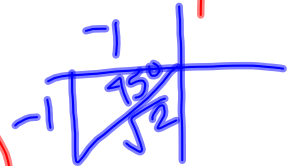
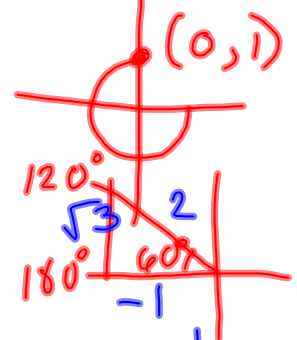


$\sec(-270^\circ)$  undefined

$\cot(120^\circ) = -\frac{1}{\sqrt{3}}$

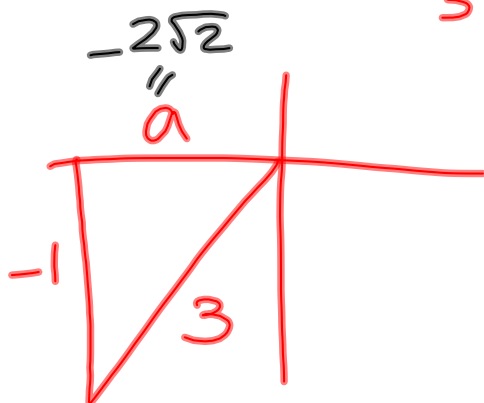
$\csc(-135^\circ) = -\sqrt{2}$

$\tan(540^\circ) = 0$



Homework questions?

33.  $\sin \theta = -\frac{1}{3}$ , Q III



$a^2 + (-1)^2 = 3^2$

$a^2 = 9 - 1 = 8$

$a = 2\sqrt{2}$

## 5.4 Radians

The circumference of a circle of radius  $r$  is given by the equation:

$$C = 2\pi r$$

Therefore, the unit circle, which has radius 1, has circumference:

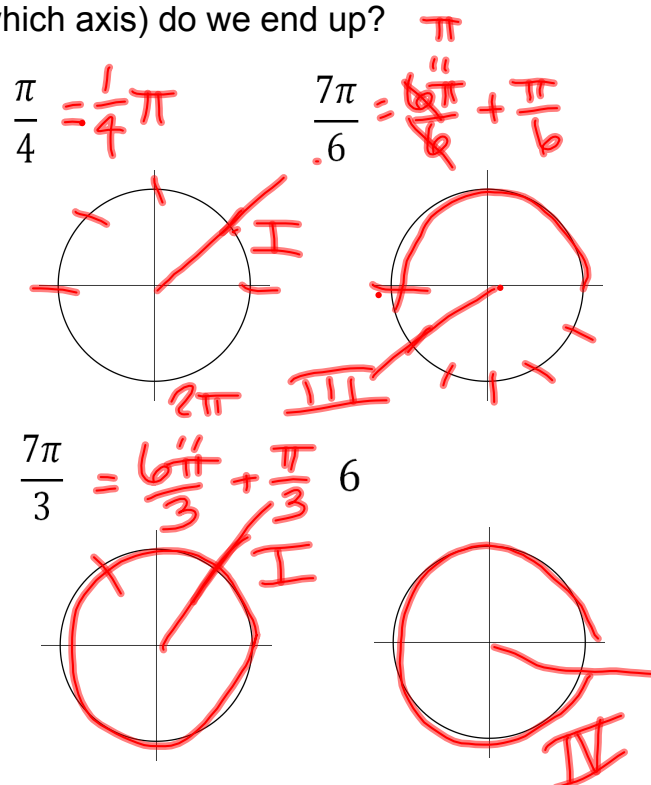
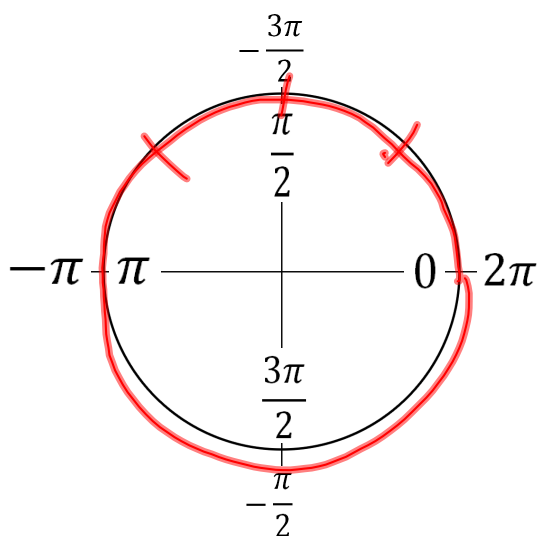
$$2\pi$$

The irrational number pi is approximately:  $\pi \approx 3.14$

Therefore  $2\pi \approx 6.28$

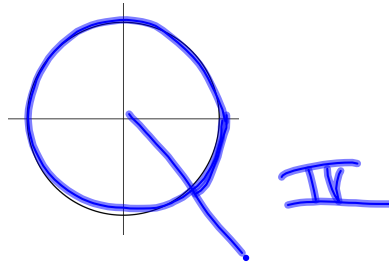
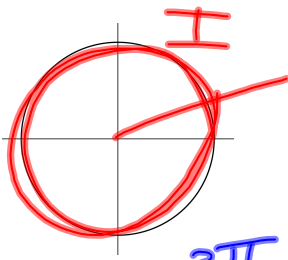
$$4\pi \approx 12.56$$

If we think about these numbers as corresponding to arc lengths around the unit circle, in which quadrant (or on which axis) do we end up?



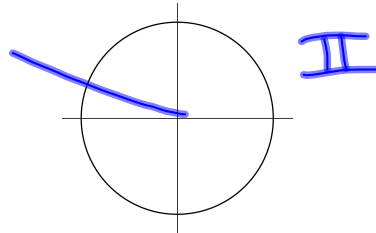
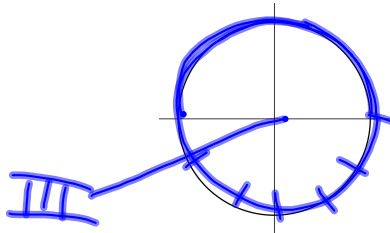
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$$-\frac{9\pi}{4} = -\frac{8\pi}{4} - \frac{\pi}{4} = -2\pi - \frac{\pi}{4}$$



$$\frac{19\pi}{6} = \frac{18\pi}{6} + \frac{\pi}{6} = 3\pi + \frac{\pi}{6}$$

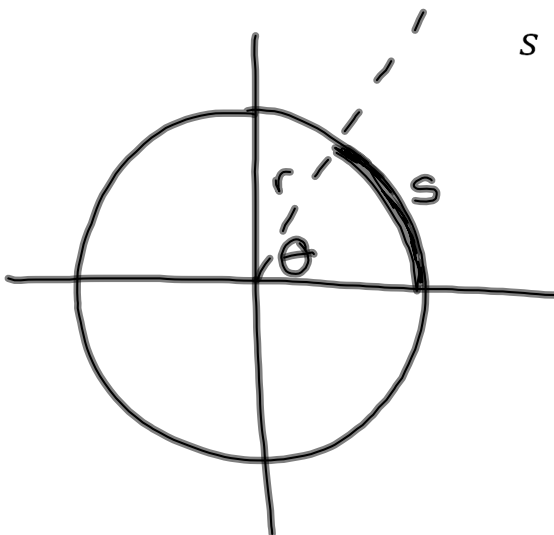
$$\frac{6\pi}{7} = \frac{6}{7}\pi$$



What is a radian?

$r$  = radius length

$s$  = arc length



When  $s = r$ , we say that the corresponding angle  $\theta$  which is subtended by arc  $s$  has measure 1 radian.

$$1 \text{ radian} \approx 57.3^\circ$$

$$\pi = 180^\circ$$

$$2\pi = 360^\circ$$

Note that  $\theta$  is independent of the radius length and any unit of measurement. Therefore radians have no associated units, and any angle measure without a degree symbol is assumed to be in radians.

Converting between radians and degrees

$$\pi = 180^\circ \quad \therefore \quad \frac{\pi}{180^\circ} = 1 = \frac{180^\circ}{\pi}$$

Convert  $225^\circ$  to radians.

$$\cancel{225^\circ} \cdot \frac{\pi}{\cancel{180^\circ}} = \boxed{\frac{5\pi}{4}}$$

Convert  $\frac{5\pi}{6}$  to degrees.

$$\frac{\cancel{5\pi}}{\cancel{6}} \cdot \frac{\cancel{180^\circ}}{\cancel{\pi}} = \boxed{150^\circ}$$

Convert  $120^\circ$  to radians.

$$120^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{2\pi}{3}}$$

Convert  $\frac{7\pi}{4}$  to degrees.

$$\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = \boxed{315^\circ}$$

Two angles in radians are:

complementary if they sum to  $\frac{\pi}{2}$ .  $(90^\circ)$

supplementary if they sum to  $\pi$ .  $(180^\circ)$

coterminal if they differ by integer multiples of  $2\pi$ .  $(360^\circ)$

Find the complement and supplement of  $\frac{5\pi}{12}$ .

comp:

$$\frac{\pi}{2} \cdot \frac{6}{6} \quad \frac{5\pi}{12} = \frac{6\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{\pi}{12}}$$

Supp:

$$\frac{12\pi}{12} - \frac{5\pi}{12} = \frac{12\pi - 5\pi}{12} = \boxed{\frac{7\pi}{12}}$$

Find one positive and one negative angle coterminal with  $-\frac{3\pi}{4}$ .

$$-\frac{3\pi}{4} + \frac{2\pi \cdot 4}{1 \cdot 4} = -\frac{3\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{5\pi}{4}}$$

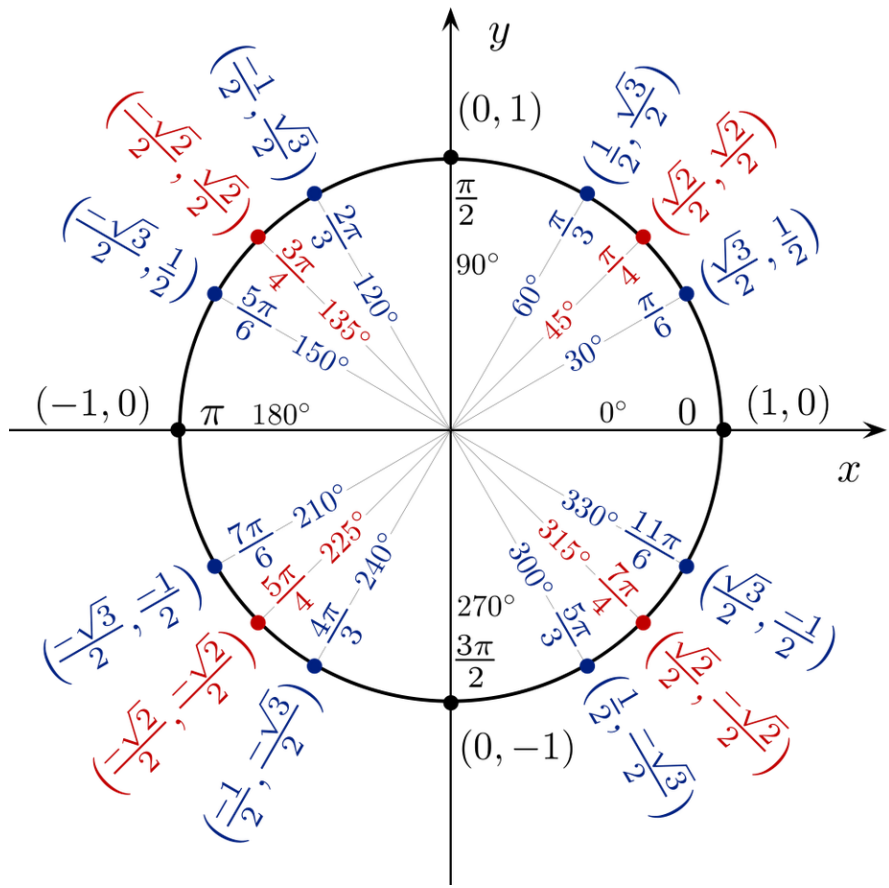
$$-\frac{3\pi}{4} - \frac{8\pi}{4} = \boxed{-\frac{11\pi}{4}}$$

Common angles:  
(memorize!)

- $\frac{\pi}{6} = 30^\circ$
- $\frac{\pi}{4} = 45^\circ$
- $\frac{\pi}{3} = 60^\circ$

Note:

- $\frac{k\pi}{6} \rightarrow 30^\circ \text{ ref. } \angle$
- $\frac{k\pi}{4} \rightarrow 45^\circ \text{ ref. } \angle$
- $\frac{k\pi}{3} \rightarrow 60^\circ \text{ ref. } \angle$
- $\frac{k\pi}{2} \rightarrow 90^\circ \text{ or } 270^\circ$
- $k\pi \rightarrow 0^\circ \text{ for } k \text{ even};$   
 $180^\circ \text{ for } k \text{ odd}$



**Homework:**

5.4

#1-7 odd - determining quadrant/location of angles in radians

#9-19 odd - compliment/supplement/coterminal angles

#21,23,27,31,45,47,53 - convert between radians and degrees

Next time:

- determine trigonometric function value of angles given in radians
- arc length/linear speed/angular speed problems