Graphing Trigonometric Functions continued...

Goal: Transform a trigonometric function of the form $y = f(x)$ to one of the form $y = af(bx + c) + d$ by observing changes in amplitude and period, as well as horizontal and vertical shifts.

Recall:

- Constants that are multiplied (divided) result in a stretching/scaling of the graph (amplitude/period changes), that we show by changing the scale on our axes
- Constants that are added (subtracted) result in shifting of the graph
- Constants outside the function (a & d) affect it vertically, as we would expect
- Constants inside the function (b & c) affect it horizontally, opposite of what we would expect

Note:

When both b and c are present (i.e. when b is anything other than 1), the horizontal shift is not just $c = \frac{c}{1}$, as it is affected by the presence of b. In this case (and in general), the horizontal shift is $\frac{c}{b}$, which we can more easily see by factoring b out in the general equation: $y = af\left[b\left(x + \frac{c}{b}\right)\right] + d$

Summary:

For a Trigonometric function of the form $\boldsymbol{y = af\left[b\left(x + \frac{c}{b}\right)\right] + d}$,

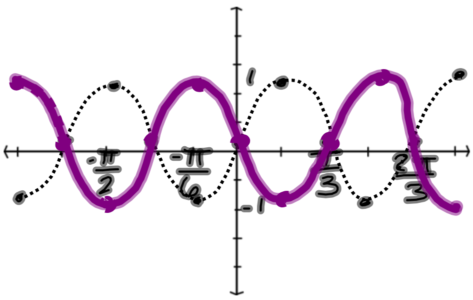
Amplitude $= |a|$ (note that amplitude is always positive)

Period $= \dfrac{original\ period\ of\ the\ function\ (\pi\ or\ 2\pi)}{|b|}$

Horizontal shift $= \dfrac{c}{b}$, $\begin{matrix} left\ if\ \frac{c}{b} > 0 \\ right\ if\ \frac{c}{b} < 0 \end{matrix}$

Vertical shift $= d$, $\begin{matrix} up\ if\ d > 0 \\ down\ if\ d < 0 \end{matrix}$

$y = \sin(3x + \pi) = \sin 3(x + \pi/3)$



amp: 1

per: $\frac{2\pi}{3}$

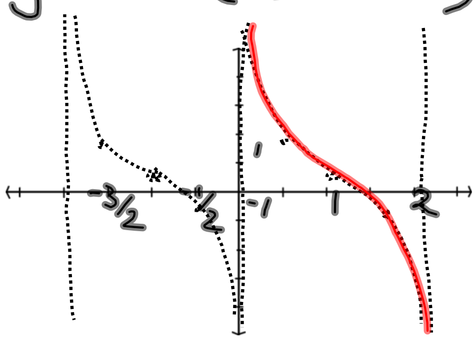
h.shift: left $\pi/3$

v.shift: none

✱ $y = -\sin 3x$

$y = \cot\left(\frac{\pi}{2}x - \pi\right) = \cot \frac{\pi}{2}(x - 2)$

$\frac{\pi}{\pi/2} = \cancel{\pi} \cdot \frac{2}{\cancel{\pi}} = 2$



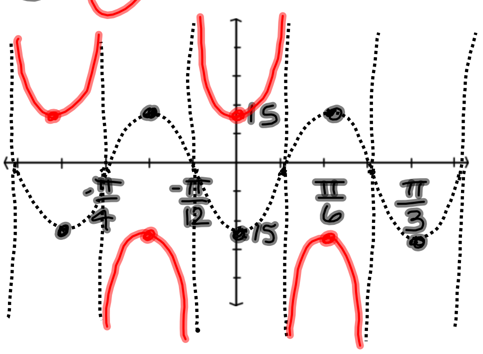
amp: 1

per: $\frac{\pi}{\pi/2} = 2$

h.shift: right 2

v.shift: none

$y = -15\sec(6x + \pi) = -15 \sec 6\left(x + \frac{\pi}{6}\right)$



amp: 15

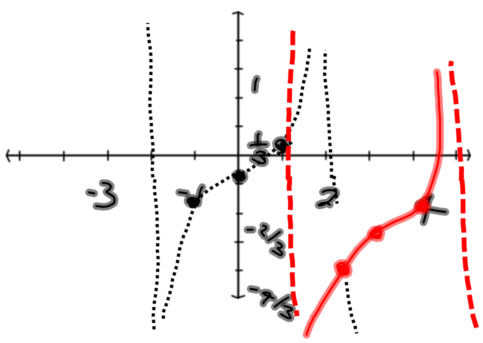
per: $\frac{2\pi}{6} = \frac{\pi}{3}$

h.shift: left $\frac{\pi}{6}$

v.shift: none

✱ $y = 15\sec 6x$

$y = \frac{1}{3}\tan\left(\frac{\pi}{4}x - \frac{3\pi}{4}\right) - \frac{2}{3} = \frac{1}{3}\tan\frac{\pi}{4}(x - 3) - \frac{2}{3}$



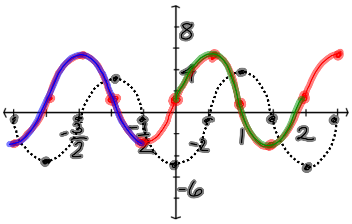
amp: 1/3

per: $\frac{\pi}{\pi/4} = 4$

h.shift: right 3

v.shift: down 2/3

$y = -4\cos\left(\pi x - \frac{3\pi}{2}\right) + 2 = -4\cos\pi\left(x - \frac{3}{2}\right) + 2$



amp: 4

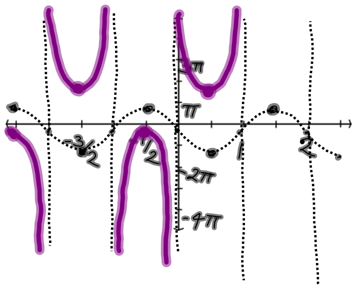
per: 2

h.shift: right 3/2

v.shift: up 2

* $y = 4\sin\pi x + 2$

$y = -\pi\csc(\pi x + \pi) + \pi = -\pi\csc\pi(x+1) + \pi$



amp: π

per: 2

h.shift: left 1

v.shift: up π

* $y = \pi\csc\pi x + \pi$

range: $(-\infty, 0] \cup [2\pi, \infty)$

HW: through #57