Sum and Difference Identities (6.1-book, 6.2-handout)

$\sin(a+b) \neq \sin a + \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

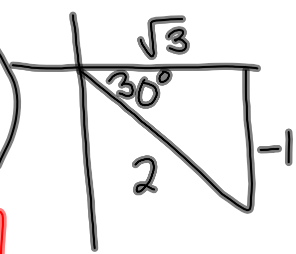
$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$; $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

6.2 handout

2. $\sin 375° = \sin(330° + 45°) =$

$= \sin 330° \cos 45° + \cos 330° \sin 45° =$

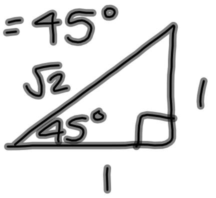
$= \left(\dfrac{-1}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right) + \left(\dfrac{\sqrt{3}}{2}\right)\left(\dfrac{1}{\sqrt{2}}\right)$



$= \boxed{\dfrac{-1+\sqrt{3}}{2\sqrt{2}}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{-\sqrt{2}+\sqrt{6}}{4}}$

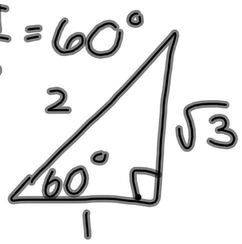
10. $\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{3}+\sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$\frac{\pi}{4}=45°$



$$=\left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)+\left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$\frac{\pi}{3}=60°$



$$=\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}=\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

14. $\sin \underbrace{167°}_{a} \cos \underbrace{107°}_{b} - \cos \underbrace{167°}_{a} \sin \underbrace{107°}_{b}$

$$=\sin(167°-107°)=\sin 60°=\boxed{\frac{\sqrt{3}}{2}}$$

20. $\sin x\cos 3x+\cos x\sin 3x$

$=\sin(x+3x)$

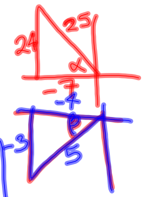
$=\boxed{\sin 4x}$

34. Given $\sin\alpha = \frac{24}{25}$, $\alpha \in Q\,II$

$\cos\beta = \frac{-4}{5}$, $\beta \in Q\,III$

Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$$



$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{28}{125} - \frac{72}{125} = \boxed{\frac{-44}{125}}$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\frac{-117}{125}}{\frac{-44}{125}} = \frac{-117}{125}\cdot\frac{125}{-44} = \boxed{\frac{117}{44}}$$

$\boxed{\alpha-\beta \text{ is in } Q\,III}$ (because both $\sin(\alpha-\beta)$ and $\cos(\alpha-\beta)$ are negative) (-,-)

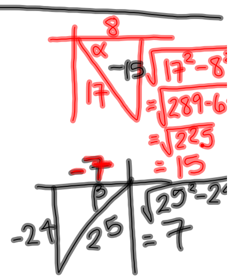
40. Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$

$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{105-192}{425} = \boxed{\frac{-87}{425}}$$

$$\begin{array}{r} \overset{3}{2}5 \\ \times 17 \\ \hline 175 \\ 250 \\ \hline 425 \end{array}$$



$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{-56-360}{425} = \boxed{\frac{-416}{425}}$$

$$\begin{array}{r} \overset{2}{2}4 \\ \times 15 \\ \hline 120 \\ 240 \\ \hline 360 \end{array}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{\frac{87}{416}}$$

$\alpha+\beta \in \boxed{Q\,III}$

6.2 handout
homework:

#1-23 odd, 35-41 odd

On Monday, Quiz on identities
& HW Check 6.1 #1-69 odd;
& 6.2 probs