Sum and Difference Identities (6.1-book, 6.2-handout)

$\sin(a+b) \neq \sin a + \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

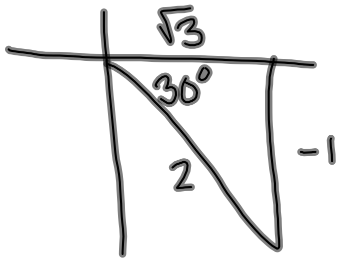
$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$\tan(a+b) = \dfrac{\tan a + \tan b}{1 - \tan a \tan b}$; $\tan(a-b) = \dfrac{\tan a - \tan b}{1 + \tan a \tan b}$

6.2 handout

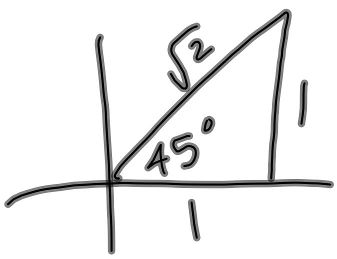
2. $\sin 375° = \sin(330° + 45°)$

$= \sin 330° \cos 45° + \cos 330° \sin 45°$



$= \left(\frac{-1}{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{\sqrt{2}}\right)$

$= \boxed{\dfrac{-1+\sqrt{3}}{2\sqrt{2}}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{-\sqrt{2}+\sqrt{6}}{4}}$



10. $\cos\left(\frac{\pi}{4}-\frac{\pi}{3}\right) =$

$\frac{\pi}{4} = 45°$

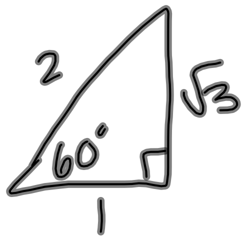
$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

$\frac{\pi}{3} = 60°$



14. $\sin \overbrace{167°}^{a} \cos \overbrace{107°}^{b} - \cos \overbrace{167°}^{a} \sin \overbrace{107°}^{b}$

$= \sin(167° - 107°) = \sin 60° = \boxed{\frac{\sqrt{3}}{2}}$

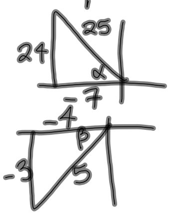
20. $\sin x \cos 3x + \cos x \sin 3x$

$= \sin(x + 3x) = \boxed{\sin 4x}$

(34) Given $\sin\alpha = \frac{24}{25}$, $\alpha \in Q\,II$

$\cos\beta = \frac{-4}{5}$, $\beta \in Q\,III$

Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.



$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$
$$= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{-96-21}{125} = \boxed{\frac{-117}{125}}$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$
$$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{28-72}{125} = \boxed{\frac{-44}{125}}$$

$$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{-117/125}{-44/125} = \frac{-117}{125}\cdot\frac{125}{-44} = \boxed{\frac{117}{44}}$$

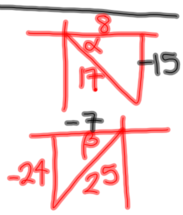
$\alpha-\beta$ is in Quadrant $\boxed{III}$ (-,-)

because $\sin(\alpha-\beta)$ & $\cos(\alpha-\beta)$ are negative

40. Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$

$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{105-192}{425} = \boxed{\frac{-87}{425}}$$

$\sqrt{17^2-8^2}$
$=\sqrt{289-64}$
$=\sqrt{225} = 15$

$\sqrt{25^2-(-24)^2}$
$=7$

17×25: 85, 340, 425

$15 \times 7 = 105$

$24 \times 8 = 192$

24×15: 120, 240, 360

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right)$$
$$= \frac{-56-360}{425} = \boxed{\frac{-416}{425}}$$

$$\tan(\alpha+\beta) = \boxed{\frac{87}{416}}$$

$\boxed{\alpha+\beta \in Q\,III}$

6.2 handout
homework:

#1-23 odd, 35-41 odd

On Monday, Quiz on identities
& HW Check 6.1 #1-69 odd;
& 6.2 probs