Cofunction Identities

$$\cos\left(\frac{\pi}{2}-x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$

$$= 0 \cdot \cos x + 1 \cdot \sin x$$

$$= \boxed{\sin x}$$

Double-Angle Identities

$$\sin(2x) = \sin(x+x)$$

$$= \sin x \cos x + \cos x \sin x$$

$$\boxed{\sin 2x = 2\sin x \cos x}$$

sin of twice any angle = 2 times the sin of that angle times cos of that angle

$$\sin 6x =$$

$$= \sin 2(3x) = 2\sin 3x \cos 3x$$

$$\sin 8x = \sin 2(4x) =$$

$$= 2\sin 4x \cos 4x$$

$$\sin 26x = \sin 2(13x)$$

$$= 2\sin 13x \cos 13x$$

$$\sin 3x = \sin 2\left(\frac{3}{2}x\right)$$

$$= 2\sin\frac{3}{2}x \cos\frac{3}{2}x$$

$$= \sin(x+2x)$$

$$\cos 2x = \cos(x+x)$$
$$= \cos x \cos x - \sin x \sin x$$
$$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$$
$$= \cos^2 x - (1 - \cos^2 x)$$
$$\boxed{\cos 2x = 2\cos^2 x - 1}$$
$$= 2(1 - \sin^2 x) - 1$$
$$= 2 - 2\sin^2 x - 1$$
$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

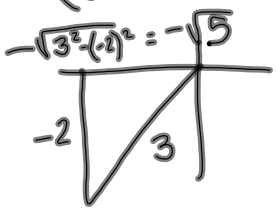
$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$
$\cos^2 x = 1 - \sin^2 x$

$$\tan 2x = \tan(x+x)$$
$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$
$$\boxed{\tan 2x = \frac{2\tan x}{1 - \tan^2 x}}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in$ QIII,
Find $\sin 2\theta, \cos 2\theta, \tan 2\theta$ & Quadrant.

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \boxed{\frac{4\sqrt{5}}{9}}$



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{4\sqrt{5}/9}{1/9} = \boxed{4\sqrt{5}}$

2θ is in Quadrant I

Half-Angle Identities

$\sin\left(\frac{x}{2}\right) = ?$

$\cos 2x = 2\cos^2 x - 1$, $\cos 2x = 1 - 2\sin^2 x$

Let $u = 2x$

$\frac{u}{2} = x$

$\cos u = 2\cos^2\left(\frac{u}{2}\right) - 1$

$\cos u = 1 - 2\sin^2\left(\frac{u}{2}\right)$

$\frac{\cos u + 1}{2} = \frac{2\cos^2\left(\frac{u}{2}\right)}{2}$

$2\sin^2\left(\frac{u}{2}\right) = 1 - \cos u$

$\sin\left(\frac{u}{2}\right) = \pm\sqrt{\frac{1-\cos u}{2}}$

$\pm\sqrt{\frac{\cos u + 1}{2}} = \sqrt{\cos^2\left(\frac{u}{2}\right)}$

$\pm\sqrt{\frac{\cos u + 1}{2}} = \cos\frac{u}{2}$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

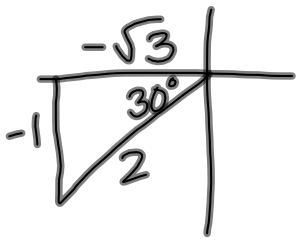
$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

$$= \boxed{\frac{1-\cos\theta}{\sin\theta}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{6}}{2}$$



$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\cdot\frac{-2}{1}$$

$$= \boxed{-2-\sqrt{3}}$$

6.3 handout:

#1-24, 30-36