Cofunction Identities

$\cos\left(\frac{\pi}{2}-x\right)=$

$=\cos\frac{\pi}{2}\cos x+\sin\frac{\pi}{2}\sin x=$

$=0\cdot\cos x+1\cdot\sin x=$

$=\sin x$

Double-Angle Identities

$\sin(2x)=\sin(x+x)=$

$=\sin x\cos x+\cos x\sin x$

$\sin 2x=2\sin x\cos x$

the sin of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 6x=\sin 2(3x)$

$=2\sin 3x\cos 3x$

$\sin 8x=\sin 2(4x)$

$=2\sin 4x\cos 4x$

$\sin 26x=\sin 2(13x)$

$=2\sin 13x\cos 13x$

$\sin 3x=\sin 2\left(\frac{3x}{2}\right)$

$=2\sin\frac{3x}{2}\cos\frac{3x}{2}$

$=\sin(2x+x)$

$$\cos 2x = \cos(x+x)$$

$$= \cos x \cos x - \sin x \sin x$$

$$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$\boxed{\cos 2x = 2\cos^2 x - 1}$$

$$= 2(1 - \sin^2 x) - 1$$

$$= 2 - 2\sin^2 x - 1$$

$$\boxed{\cos 2x = 1 - 2\sin^2 x}$$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

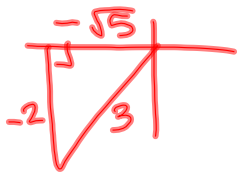
$$\tan 2x = \tan(x+x)$$

$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$\boxed{\tan 2x = \frac{2\tan x}{1 - \tan^2 x}}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in$ QIII,
Find $\sin 2\theta, \cos 2\theta, \tan 2\theta$ & Quadrant.

$\sin 2\theta = 2\sin\theta\cos\theta$
$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \boxed{\frac{4\sqrt{5}}{9}}$



$\sqrt{3^2-(-2)^2} = \sqrt{5}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$
$= \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{4\sqrt{5}/9}{1/9} = \boxed{4\sqrt{5}}$

2θ is in quadrant I

Half-Angle Identities

$\sin\left(\frac{x}{2}\right) = ?$

$\cos 2x = 2\cos^2 x - 1$, $\cos 2x = 1 - 2\sin^2 x$

Let $x = \frac{\theta}{2}$

$\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

$\cos\theta + 1 = 2\cos^2\frac{\theta}{2}$

$\frac{\cos\theta + 1}{2} = \cos^2\frac{\theta}{2}$

$\pm\sqrt{\frac{\cos\theta+1}{2}} = \cos\frac{\theta}{2}$

$x = \frac{\theta}{2}$

$\cos\theta = 1 - 2\sin^2\frac{\theta}{2}$

$2\sin^2\frac{\theta}{2} = 1 - \cos\theta$

$\sin^2\frac{\theta}{2} = \frac{1-\cos\theta}{2}$

$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$$\tan\frac{\theta}{2} = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

$$= \frac{\sin\theta}{1+\cos\theta}$$

$$= \boxed{\frac{1-\cos\theta}{\sin\theta}}$$

$$\tan\frac{7\pi}{12} = \tan\frac{7\pi/6}{2}$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

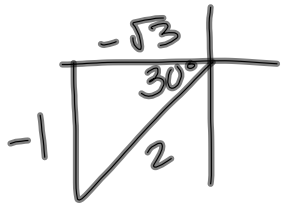
$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}}$$

$$= \left(1+\frac{\sqrt{3}}{2}\right)\cdot\frac{-2}{1}$$

$$= \boxed{-2-\sqrt{3}}$$

$$\frac{7\pi}{12} = \frac{1}{2}\theta$$

$$\frac{7\pi}{6} = \theta$$



6.3 handout:

#1-24, 30-36