

6.3


$$13. \cos 157.5^\circ = \cos \frac{157.5^\circ \times 2}{2} = \cos \frac{x}{2} =$$

$$= \cos \frac{315^\circ}{2} = -\sqrt{\frac{1 + \cos 315^\circ}{2}}$$

$$= -\sqrt{\frac{\frac{2}{2} \cdot 1 + \frac{\sqrt{2}}{2}}{2}} = -\sqrt{\left(\frac{2 + \sqrt{2}}{2}\right) \cdot \frac{1}{2}}$$

$$= -\sqrt{\frac{2 + \sqrt{2}}{4}} = \frac{-\sqrt{2 + \sqrt{2}}}{2}$$

$\cos \frac{x}{2} =$
 $+\sqrt{\frac{1 + \cos x}{2}}$
 $-\sqrt{\frac{1 + \cos x}{2}}$



$$\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

6.3 Handout Prove the identity.

$$50. \cos 8x = \cos^2 4x - \sin^2 4x$$

$$\text{LHS} = \cos 2(4x) = \cos^2 4x - \sin^2 4x = \text{RHS} \checkmark$$

$$52. \frac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$$

$$\text{LHS} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} =$$

$$= \cot^2 x - 1 = \text{RHS} \checkmark$$

$$54. \frac{1}{1-\cos 2x} = \frac{1}{2} \csc^2 x$$

$$\text{LHS} = \frac{1}{1-(1-2\sin^2 x)} = \frac{1}{2\sin^2 x} = \frac{1}{2} \csc^2 x = \text{RHS}$$

$$56. \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = \cot 2x$$

$$\text{RHS} = \frac{\cos 2x}{\sin 2x} = \frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = \text{LHS}$$

$$60. \sin 2x - \cot x = -\cot x \cos 2x$$

$$\begin{aligned} \text{RHS} &= -\cot x (1 - 2\sin^2 x) \\ &= -\cot x + 2\cot x \sin^2 x \\ &= -\cot x + 2 \cdot \frac{\cos x}{\sin x} \cdot \sin^2 x \\ &= -\cot x + 2\sin x \cos x \\ &= -\cot x + \sin 2x \\ &= \text{LHS} \end{aligned}$$

$$62. \sin 4x = 4\sin x \cos^3 x - 4\cos x \sin^3 x$$

$$\text{LHS} = \sin^2(2x) = 2\sin 2x \cos 2x =$$

$$= \underbrace{2(2\sin x \cos x)}_{4\sin x \cos x} (\cos^2 x - \sin^2 x) =$$

$$= 4\sin x \cos^3 x - 4\sin^3 x \cos x$$

$$= \text{RHS}$$

$$64. 2\cos^4 x - \cos^2 x - 2\sin^2 x \cos^2 x + \sin^2 x = \cos^2 2x$$

$$\text{RHS} = (\cos 2x)^2 = (\cos 2x)(\cos 2x) =$$

$$= (\cos^2 x - \sin^2 x)(2\cos^2 x - 1) =$$

$$= 2\cos^4 x - \cos^2 x - 2\sin^2 x \cos^2 x + \sin^2 x$$

$$= \text{LHS}$$

$$66. \sin 4x = 4 \sin x \cos x - 8 \cos x \sin^3 x$$

$$\text{LHS} = \sin 2(2x) = 2 \sin 2x \cos 2x =$$

$$= 2 \underbrace{(2 \sin x \cos x)}_{4 \sin x \cos x} \underbrace{(1 - 2 \sin^2 x)} =$$

$$= 4 \sin x \cos x - 8 \sin^3 x \cos x$$

$$= \text{RHS}$$

$$68. \sin 3x + \sin x = 4 \sin x - 4 \sin^3 x$$

$$\text{LHS} = \sin(2x+x) + \sin x =$$

$$= \sin 2x \cos x + \cos 2x \sin x + \sin x =$$

$$= (2 \sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x + \sin x =$$

$$= 2 \sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x + \sin x =$$

$$= 3 \sin x \cos^2 x - \sin^3 x + \sin x =$$

$$= 3 \sin x (1 - \sin^2 x) - \sin^3 x + \sin x =$$

$$= 3 \sin x - 3 \sin^3 x - \sin^3 x + \sin x =$$

$$= 4 \sin x - 4 \sin^3 x =$$

$$= \text{RHS}$$

6.3 handout :

49-93 odd