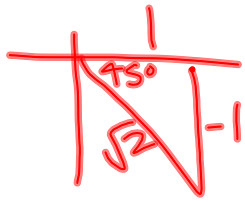
6.3

13. $\cos 157.5^\circ = \cos\frac{157.5^\circ \times 2}{2} =$

$\cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$

$= \cos\frac{315^\circ}{2} = -\sqrt{\frac{1+\cos 315^\circ}{2}}$



$= -\sqrt{\frac{\frac{2}{2}\cdot 1+\frac{\sqrt{2}}{2}}{\frac{2}{1}}} = -\sqrt{\left(\frac{2+\sqrt{2}}{2}\right)\cdot\frac{1}{2}}$

$\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$= -\sqrt{\frac{2+\sqrt{2}}{4}} = \boxed{\frac{-\sqrt{2+\sqrt{2}}}{2}}$

6.3 Handout Prove the identity.

50. $\cos 8x = \cos^2 4x - \sin^2 4x$

LHS $= \cos 2(4x) = \cos^2 4x - \sin^2 4x =$ RHS

52. $\frac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

LHS $= \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} =$

$= \cot^2 x - 1 =$ RHS

54. $\dfrac{1}{1-\cos 2x} = \dfrac{1}{2}\csc^2 x$

$LHS = \dfrac{1}{1-(1-2\sin^2 x)} = \dfrac{1}{2\sin^2 x} = \dfrac{1}{2}\csc^2 x = RHS$

$1-1+2\sin^2 x$ $\quad \dfrac{1}{2}\cdot\dfrac{1}{\sin^2 x}$

$\dfrac{ab}{cd} = a\cdot\dfrac{b}{cd} = \dfrac{b}{d}\cdot\dfrac{a}{c}$

56. $\dfrac{\cos^2 x-\sin^2 x}{2\sin x\cos x} = \cot 2x$

$RHS = \dfrac{\cos 2x}{\sin 2x} = \dfrac{\cos^2 x-\sin^2 x}{2\sin x\cos x} = LHS$

60. $\sin 2x - \cot x = -\cot x\cos 2x$

$LHS = 2\sin x\cos x - \cot x$

$= \dfrac{2\sin x\cos x}{1}\cdot\dfrac{\sin x}{\sin x} - \cot x$

$= \dfrac{2\sin^2 x}{1}\cdot\dfrac{\cos x}{\sin x} - \cot x$

$= 2\sin^2 x\cdot\cot x - \cot x$

$= -\cot x(-2\sin^2 x + 1)$

$= -\cot x(1-2\sin^2 x)$

$= -\cot x\cos 2x = RHS$ ✓

62. $\sin 4x = 4\sin x\cos^3 x - 4\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x =$

$= \underbrace{2(2\sin x\cos x)}_{4\sin x\cos x}(\cos^2 x - \sin^2 x) =$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x =$

$= RHS$

64. $2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x = \cos^2 2x$

$RHS = (\cos 2x)^2 = (\cos 2x)\cdot(\cos 2x) =$

$= (2\cos^2 x - 1)(\cos^2 x - \sin^2 x) =$

$= 2\cos^4 x - 2\sin^2 x\cos^2 x - \cos^2 x + \sin^2 x =$

$= LHS$

66. $\sin 4x = 4\sin x\cos x - 8\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x =$

$= \underbrace{2(2\sin x\cos x)}_{4\sin x\cos x}(1-2\sin^2 x) =$

$= 4\sin x\cos x - 8\sin^3 x\cos x$

$= RHS$

68. $\sin 3x + \sin x = 4\sin x - 4\sin^3 x$

$LHS = \sin(2x+x) + \sin x$

$= \sin 2x\cos x + \cos 2x\sin x \quad + \sin x$

$= (2\sin x\cos x)\cdot\cos x + (\cos^2 x - \sin^2 x)\cdot\sin x + \sin x$

$= 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x + \sin x$

$= 3\sin x\cos^2 x - \sin^3 x + \sin x$

$= 3\sin x(1-\sin^2 x) - \sin^3 x + \sin x$

$= 3\sin x - 3\sin^3 x - \sin^3 x + \sin x$

$= 4\sin x - 4\sin^3 x = RHS$

6.3 handout :

49-93 odd