6.3 handout

72. $\cos^2\frac{x}{2} = \frac{\sec x + 1}{2\sec x}$

$LHS = \left(\cos\frac{x}{2}\right)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \frac{(1+\cos x)}{2}\cdot\frac{\sec x}{\sec x}$

$= \frac{\sec x + \cos x \sec x}{2\sec x} = \frac{\sec x + \cos x \cdot \frac{1}{\cos x}}{2\sec x}$

$= \frac{\sec x + 1}{2\sec x} = RHS$ ✓

76. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$

$LHS = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 - \left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$

$= \frac{1+\cos x}{2} - \frac{1-\cos x}{2}$

$= \frac{2\cos x}{2} = \cos x = RHS$ ✓

Alt. Proof:

$LHS = \cos 2\left(\frac{x}{2}\right) = \cos x = RHS$

86. $\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$

$LHS = \frac{1-2\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{2\sin^2 x}{\sin^2 x}$

$= \csc^2 x - 2 = RHS$ ✓

Alt: $LHS = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} =$

$= \cot^2 x - 1$

$= \csc^2 x - 1 - 1$

$= \csc^2 x - 2 = RHS$

$\cot^2 x + 1 = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

88. $\frac{2\cos 2x}{\sin 2x} = \cot x - \tan x$

$LHS = \frac{2(\cos^2 x - \sin^2 x)}{2\sin x \cos x} =$

$= \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$

$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x = RHS$