

Inverse Trigonometric Functions

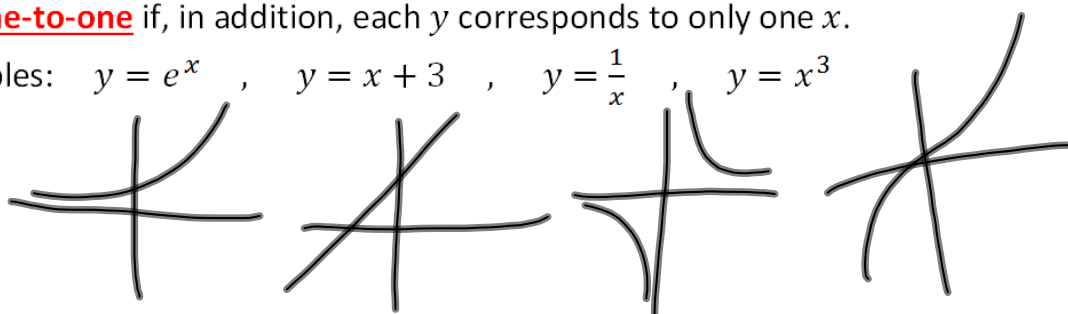
(6.4 book / 6.5 handout)

Recall from Algebra:

- f is a **function** if each input value (x) has a unique output $f(x)$.

Examples: $f(x) = x^2 - 2$, $f(x) = \sqrt{x}$ 

- f is **one-to-one** if, in addition, each y corresponds to only one x .

Examples: $y = e^x$, $y = x + 3$, $y = \frac{1}{x}$, $y = x^3$ 

- If f is a one-to-one function, we can define its inverse $f^{-1}(x)$.
Note that this notation is not exponentiation, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$
- $f(x)$ and $g(x)$ are **inverses** if
 $(f \circ g)(x) = f(g(x)) = x = g(f(x)) = (g \circ f)(x)$,
that is, **inverse functions "undo" each other.**

Example: $f(x) = x^3$, $g(x) = \sqrt[3]{x}$

$$(f \circ g)(x) = f(g(x)) = (\sqrt[3]{x})^3 = x$$

$$(g \circ f)(x) = g(f(x)) = \sqrt[3]{x^3} = x$$

What do we mean by an Inverse Trig function?

Recall that **for a basic Trigonometric function**, e.g. $f(x) = \sin x$,

- The input (x) is an angle
- The output $f(x)$ is a ratio of sides

So **for an inverse Trigonometric function**,

- The input (x) is a ratio of sides
- The output $f(x)$ is an angle

Construction of the inverse of $f(x) = \sin x$:

$$y = \sin x$$

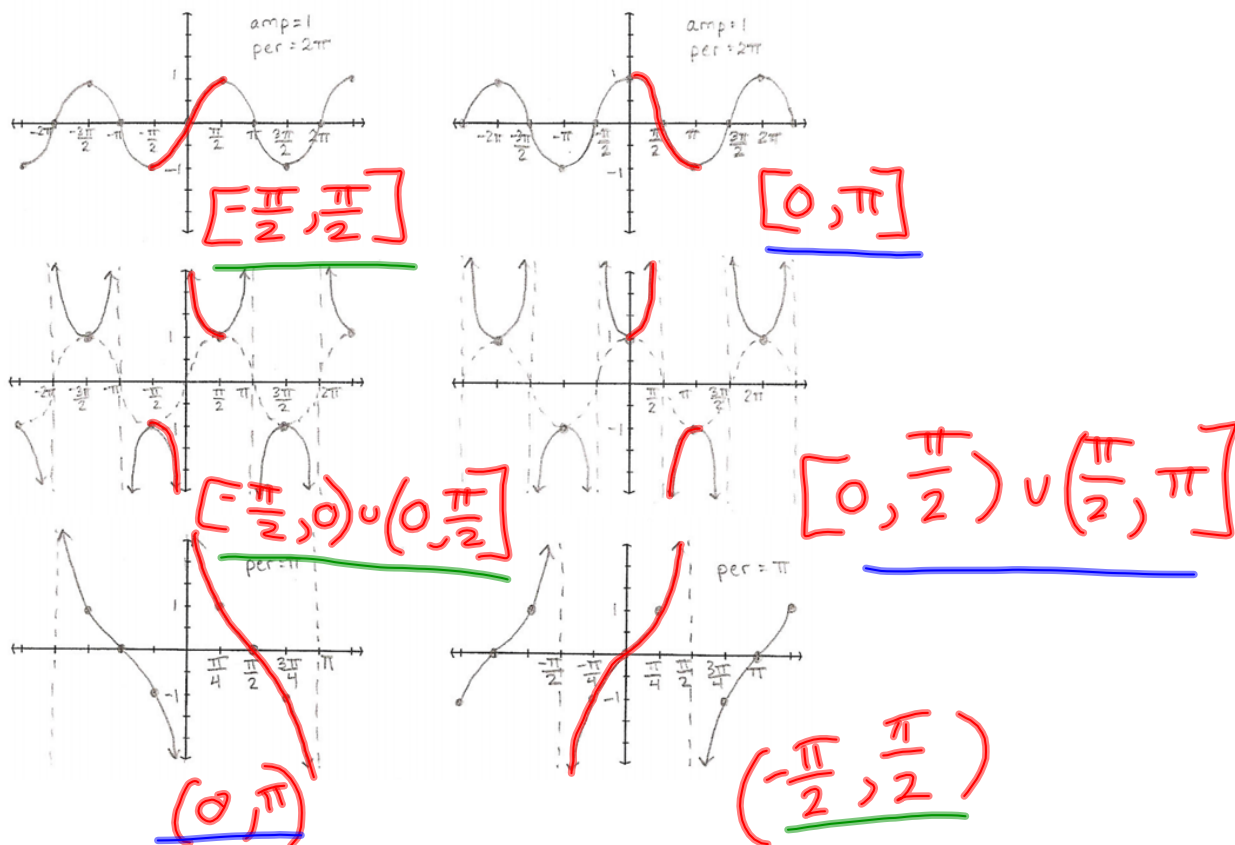
$$x = \sin y$$

$y = \text{the angle whose sine value is } x$

$$f^{-1}(x) = \sin^{-1}(x) \quad \text{or} \quad f^{-1}(x) = \arcsin x$$

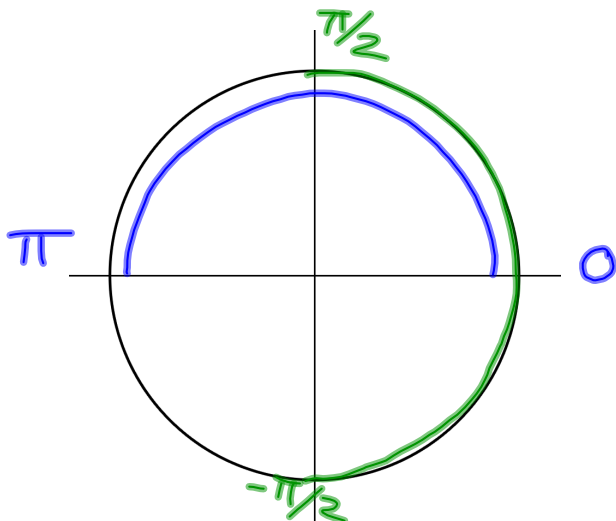
$$\star \sin^{-1}(x) \neq \frac{1}{\sin x}$$

But Trigonometric functions aren't one-to-one – how is the inverse defined? We must restrict the domain!



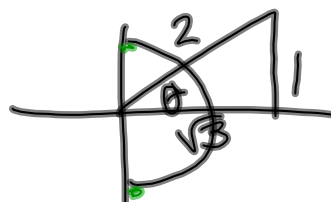
Summary of Restricted Domains:

Interval	Functions	Quadrants
$(-\frac{\pi}{2}, \frac{\pi}{2})$	$\sin x, \csc x, \tan x$	IV & I
$(0, \pi)$	$\cos x, \sec x, \cot x$	I & II



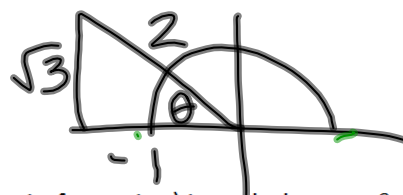
Evaluate the inverse trigonometric expression.

$\sin^{-1}(\frac{1}{2}) = 30^\circ = \frac{\pi}{6}$



In words: What angle θ , between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (the restricted domain for sine) is such that $\sin \theta = \frac{1}{2}$?

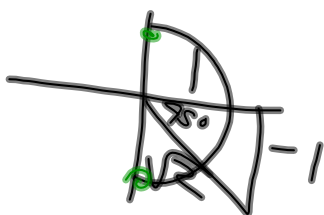
$\cos^{-1}(-\frac{1}{2}) = 120^\circ = \frac{2\pi}{3}$



In words: What angle θ , between 0 and π (the restricted domain for cosine) is such that $\cos \theta = -\frac{1}{2}$?

$\tan^{-1}(-1) = -\frac{\pi}{4} = -75^\circ$

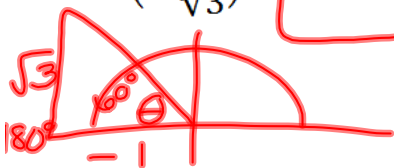
What angle θ betw $-\frac{\pi}{2}$ & $\frac{\pi}{2}$ is s.t. $\tan \theta = -1$?



$315^\circ / \frac{7\pi}{4}$ is not between $-\frac{\pi}{2}$ & $\frac{\pi}{2}$

Evaluate.

1. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 120^\circ = \frac{2\pi}{3}$



2. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = 60^\circ = \frac{\pi}{3}$



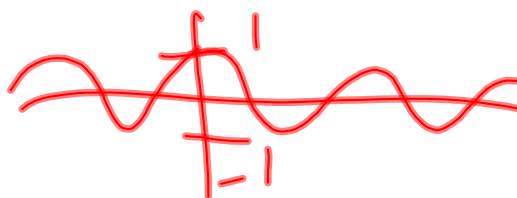
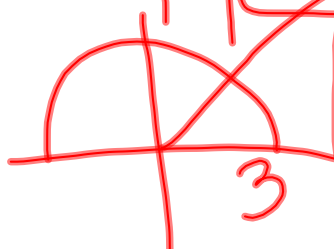
3. $\csc^{-1}(-2) = -30^\circ = -\frac{\pi}{6}$



4. $\tan^{-1}(0) = 0^\circ = 0$



5. $\cos^{-1}(3) = \text{undefined}$



What happens when we compose a Trigonometric function with its inverse?

According to the definition,

$f(x)$ and $g(x)$ are inverses if $f(g(x)) = x$ and $g(f(x)) = x$
(for all x -values in the respective domains of g and f)

We would then expect

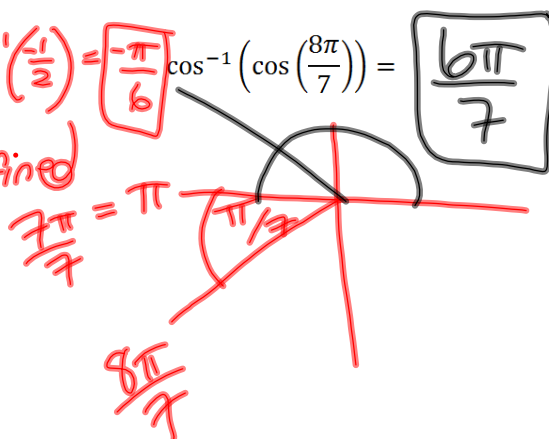
$\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin x) = x$



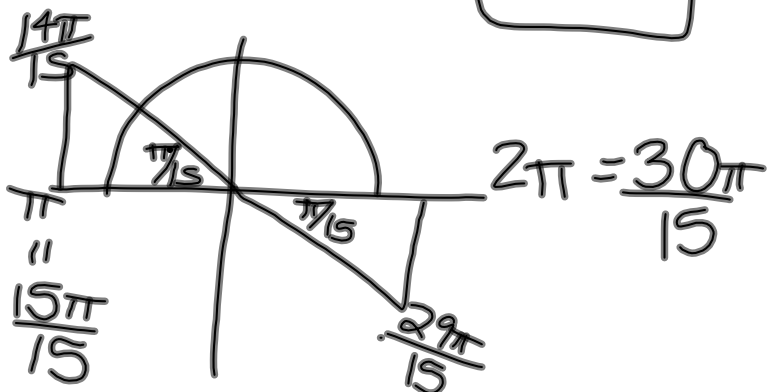
$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$ $\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$ $\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) = \frac{6\pi}{7}$

$\sin(\sin^{-1} 3) = \text{undefined}$



$$\cot^{-1}\left(\cot\frac{29\pi}{15}\right) = \boxed{\frac{14\pi}{15}}$$



Homework:

6.5 Handout #1-24

ALL