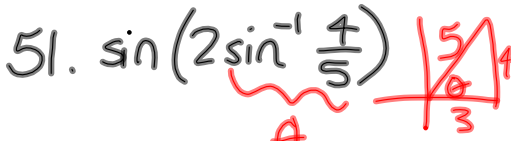
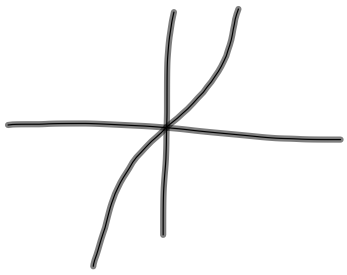
Homework questions?

51. $\sin\left(2\sin^{-1}\frac{4}{5}\right)$



$= \sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \cdot \frac{4}{5} \cdot \frac{3}{5} = \boxed{\frac{24}{25}}$



range of $\tan x$: $(-\infty, \infty)$

47.

domain of $\sin^{-1}$ & $\cos^{-1}$ is $[-1, 1]$

~~45. $\cos\left(\sin^{-1}\frac{7}{25}\right)$~~

49. $\cos\left(2\sin^{-1}\frac{\sqrt{2}}{2}\right)$

$= \cos\left(2\sin^{-1}\frac{1}{\sqrt{2}}\right) = \cos\left(2 \cdot \frac{\pi}{4}\right) = \cos\frac{\pi}{2} = \boxed{0}$

Solving Trigonometric Equations

$\sin^{-1}\left(\frac{1}{2}\right)$ versus $\sin x = \frac{1}{2}$

$= \frac{\pi}{6}$

only has one answer

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, -\frac{7\pi}{6}, -\frac{11\pi}{6}, \ldots$

infinitely many answers!

$$x = \frac{\pi}{6} + 2\pi k$$

$$x = \frac{5\pi}{6} + 2\pi k$$

$k \in \mathbb{Z}$ (k is an integer)

$\mathbb{R}, \mathbb{N}, \mathbb{Q}, \mathbb{Z}$

6.6 Handout

Solve for $x \in [0, 2\pi)$.

2. $\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

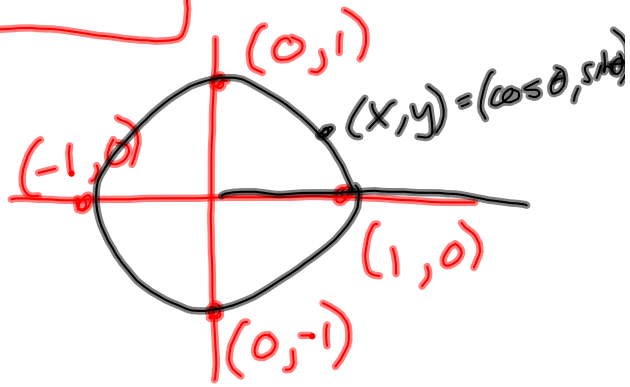
$\sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

4. $\cos x - 1 = 0$

$\cos x = 1$

$x = 0$



6. $2\sin x \cos x = \sqrt{3}\sin x$

~~$2\cos x = \sqrt{3}$~~

$\frac{x^2}{x} = \frac{x}{x}$

$x = 1$

$\longrightarrow x^2 - x = 0$

$x(x-1) = 0$

$x = 0, 1$

Algebra Review

$(x-2)(x-3)(x-4)=0$

$x-2=0,\ x-3=0,\ x-4=0$

$x=2,3,4$

$(x-2)(x+3)=5$

$x-2=5,\ x+3=5$

The **Zero Product Property** states:

If $AB=0$, then $A=0$ or $B=0$.

$x^2=9$

$x=\pm 3$

The **Square Root Theorem** states:

If $[f(x)]^2=c$, then $f(x)=\pm\sqrt{c}$

6. $2\sin x\cos x = \sqrt{3}\sin x$

$[0, 2\pi)$

$2\sin x\cos x - \sqrt{3}\sin x = 0$

$\sin x(2\cos x - \sqrt{3}) = 0$

$\sin x = 0$ $\quad 2\cos x - \sqrt{3} = 0$

$x = 0, \pi$ $\quad 2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

8. $\cos^2 x - 1 = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$(\cos x - 1)(\cos x + 1) = 0$

$\cos x = 1, \cos x = -1$

$x = 0, \pi$

10. $\sec^2 x + \sqrt{3}\sec x - \sqrt{2}\sec x - \sqrt{6} = 0$

$\sec x(\sec x + \sqrt{3}) - \sqrt{2}(\sec x + \sqrt{3}) = 0$

$(\sec x + \sqrt{3})(\sec x - \sqrt{2}) = 0$

$\sec x = -\sqrt{3}$, $\sec x = \sqrt{2}$

$x =$ 2 not-so-nice ∠'s in Q's II & III

≈ 2.1863 & 4.0969

$x = \frac{\pi}{4}, \frac{7\pi}{4}$

14. $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

let $u = \cos x$

$2u^2 + 3u + 1 = 0$

$(2u+1)(u+1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$ $\cos x = -1$

$x = \frac{2\pi}{3}, \frac{4\pi}{3}$ $x = \pi$

18. $4\cos^3 x = 3\cos x$

$4\cos^3 x - 3\cos x = 0$

$\cos x(4\cos^2 x - 3) = 0$

$\cos x = 0$, $\cos^2 x = \frac{3}{4}$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3}\tan x$

$\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$

$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$

$(\tan x + 1)(\tan x - \sqrt{3}) = 0$

$\tan x = -1$ $\tan x = \frac{\sqrt{3}}{1}$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$ $x = \frac{\pi}{3}, \frac{4\pi}{3}$

22. $\cos^4 x = \cos^2 x$

$\cos^4 x - \cos^2 x = 0$

$\cos^2 x(\cos^2 x - 1) = 0$

$\cos^2 x = 0$ $\quad$ $\cos^2 x = 1$

$\cos x = 0$ $\quad$ $\cos x = \pm 1$

$x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ $x = 0, \pi$

New Directions: Find ALL the solutions (not just in $[0,2\pi)$)

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$

$\sec 3x = \frac{2\sqrt{3}}{3}$

$\sec(3x) = \frac{2}{\sqrt{3}}$

$\sec\theta = \frac{2}{\sqrt{3}}$

$\theta = \frac{\pi}{6} + 2\pi k$

$\theta = \frac{11\pi}{6} + 2\pi k$

$3x = \frac{\pi}{6} + 2\pi k$, $3x = \frac{11\pi}{6} + 2\pi k$

$x = \frac{\pi}{18} + \frac{2\pi k}{3}$, $x = \frac{11\pi}{18} + \frac{2\pi k}{3}$

68. $\cos\left(2x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} = -\frac{1}{\sqrt{2}}$

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$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k$, $2x - \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$

$x = \frac{\pi}{2} + \pi k$

$2x = \frac{3\pi}{2} + 2\pi k$

$x = \frac{3\pi}{4} + \pi k$

Homework (from 6.6 Handout)

#1-21 odd - finding solutions in $[0, 2\pi)$

#61-69 odd - finding all possible solutions $(+2\pi k)$