

Solve for $x \in [0, 2\pi)$.

$$(\sin x - \cos x)^2 = 1^2$$

$$\sin^2 x - 2 \sin x \cos x + \cos^2 x = 1$$

$$-2 \sin x \cos x = 1$$

$$0 = \sin(2x)$$

$$2x = 0, \pi, 2\pi, 3\pi$$

$$x = \cancel{0}, \pi/2, \pi, \cancel{3\pi/2}$$

* raising both sides of an equation to an even power may introduce extraneous solutions

$$0 \leq x < 2\pi$$

$$0 \leq 2x < 4\pi$$

Check:

$$\sin 0 - \cos 0 = 0 - 1 = -1$$

$$\sin \frac{\pi}{2} - \cos \frac{\pi}{2} = 1 - 0 = 1$$

$$\sin \pi - \cos \pi = 0 - (-1) = 1$$

$$\sin \frac{3\pi}{2} - \cos \frac{3\pi}{2} = -1 - 0 = -1$$

$$\cos(4x) = \frac{1}{\sqrt{2}}$$

$$0 \leq x < 2\pi$$

$$0 \leq 4x < 8\pi$$

$$4x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4}, \frac{25\pi}{4}, \frac{31\pi}{4}$$

$$2\pi = \frac{8\pi}{4}$$

$$x = \frac{\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{15\pi}{16}, \frac{17\pi}{16}, \frac{23\pi}{16}, \frac{25\pi}{16}, \frac{31\pi}{16}$$

$$\tan(5x) = 0$$

$$5x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, 8\pi, 9\pi$$

$$x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}$$

$$72. \cos 2x = 2 \cos x - 1$$

$$2 \cos^2 x - 1 = 2 \cos x - 1$$

$$2 \cos^2 x = 2 \cos x$$

$$\cos^2 x = \cos x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x (\cos x - 1) = 0$$

$$\cos x = 0 \quad \cos x = 1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, 0$$

$$74. \sin 4x - \cos 2x = 0$$

$$\sin 2(2x)$$

$$2 \sin 2x \cos 2x - \cos 2x = 0$$

$$\cos 2x (2 \sin 2x - 1) = 0$$

$$\cos 2x = 0, \quad \sin 2x = \frac{1}{2}$$

$$2x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$$

$$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$$

$$78. \cos^a 2x \cos^b x - \sin^a 2x \sin^b x = 0$$

$$\cos(2x+x) = 0$$

$$\cos 3x = 0$$

$$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$$

$$x = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{3\pi}{2}, \frac{11\pi}{6}$$

$$82. \cos 3x + \cos x = 0$$

$$\cos(2x+x) + \cos x = 0$$

$$\cos 2x \cos x - \sin 2x \sin x + \cos x = 0$$

$$(\cos^2 x - \sin^2 x) \cos x - (2 \sin x \cos x) \sin x + \cos x = 0$$

$$\cos^3 x - \sin^2 x \cos x - 2 \sin^2 x \cos x + \cos x = 0$$

$$\cos^3 x - 3 \sin^2 x \cos x + \cos x = 0$$

$$\cos^3 x - 3 \cos x (1 - \cos^2 x) + \cos x = 0$$

$$\cos^3 x - 3 \cos x + 3 \cos^3 x + \cos x = 0$$

$$4 \cos^3 x - 2 \cos x = 0$$

$$2 \cos x (2 \cos^2 x - 1) = 0$$

$$2 \cos x = 0$$

$$\cos x = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm \frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

$$84. 2 \sin x \cos x - 2\sqrt{2} \sin x - \sqrt{3} \cos x + \sqrt{6} = 0$$

$$76. \tan \frac{x}{2} = 1 - \cos x$$

Homework:

6.6 Handout #71-83 odd;

Examples #3,4,7,8 from solving equaons handout