4. $\frac{-1}{\sqrt{2}}$

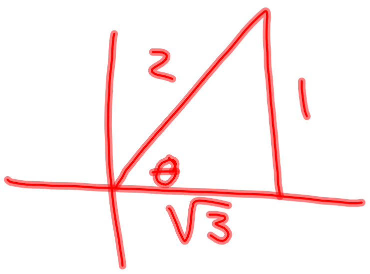
5. 1

6. $\frac{-1}{\sqrt{3}}$

7. $\frac{\pi}{6}$

8. $\frac{3\pi}{4}$

9. $\frac{-\pi}{2}$



$$LHS = \frac{\cos^2 x - \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \cot^2 x - 1$$

$$= RHS$$

Ex 4

$$\sin\left(3x + \frac{\pi}{4}\right) = 1$$

$$3x + \frac{\pi}{4} = \frac{\pi}{2} + 2\pi k$$

$$3x = \frac{\pi}{4} + 2\pi k$$

$$\boxed{x = \frac{\pi}{12} + \frac{2\pi k}{3}}$$

71. $\cos 2x = 1 - 3\sin x$ $\quad 0 \le x < 2\pi$

$$1 - 2\sin^2 x = 1 - 3\sin x$$

$$3\sin x - 2\sin^2 x = 0$$

$$\sin x\,(3 - 2\sin x) = 0$$

$\sin x = 0$ $\quad \cancel{\sin x = \frac{3}{2}}$

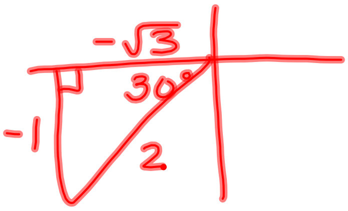
$$\boxed{x = 0, \pi}$$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{6}}{2} = \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\left(\frac{-2}{1}\right)$$



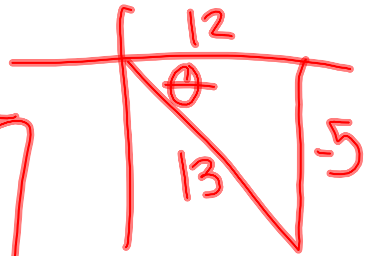
$$= \boxed{-2-\sqrt{3}}$$

2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$$= \cos(212°-122°) = \cos 90° = \boxed{0}$$

3. Find $\sin 2\theta, \cos 2\theta, and \tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\cdot\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$$



$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120/169}{119/169} = \boxed{\frac{-120}{119}}$$

What quadrant is 2θ in? IV

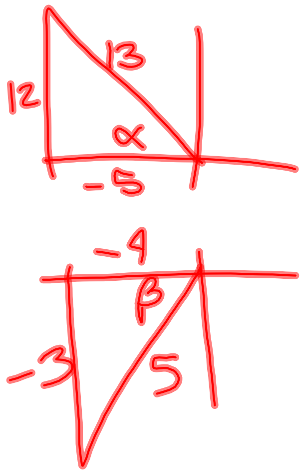
4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha+\beta)$.

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

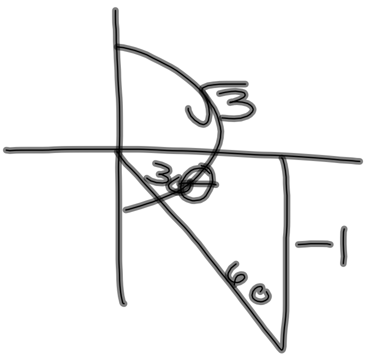
$$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$

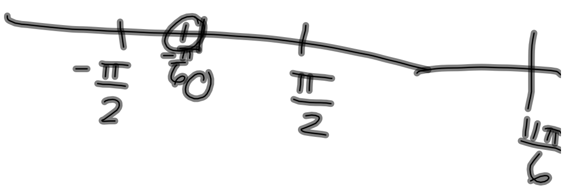
$$= \frac{-48}{65} + \frac{15}{65}$$

$$= \boxed{\frac{-33}{65}}$$



5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians. $\boxed{-\frac{\pi}{6}}$

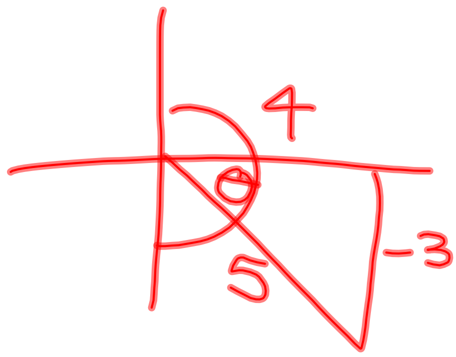




6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$

$$\sec\Big(\underbrace{\sin^{-1}\left(-\tfrac{3}{5}\right)}_{\theta}\Big)$$

$$= \boxed{\frac{5}{4}}$$



7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$$2\sin^3 x - \sin x = 0$$

$$\sin x\,(2\sin^2 x - 1) = 0$$

$\sin x = 0$

$\boxed{x = 0, \pi}$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$\boxed{x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

9. Prove the identity.

$$\frac{1 + \cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$$LHS = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} + \frac{1 - \sin^2 x}{\sin^2 x}$$

$$= \frac{1}{\sin^2 x} + \frac{1}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \csc^2 x + \csc^2 x - 1$$

$$= 2\csc^2 x - 1$$

$$= RHS \checkmark$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$

$$= \frac{1-\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x$$

$$= RHS \checkmark$$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$

$$\sin(2x+x) + \sin x - 2\sin x\cos x = 0$$

$$\sin 2x\cos x + \cos 2x\sin x + \sin x - 2\sin x\cos x = 0$$

$$2\sin x\cos x \cdot \cos x + (1-2\sin^2 x)\sin x + \sin x - 2\sin x\cos x = 0$$

$$2\sin x\cos^2 x + \sin x - 2\sin^3 x + \sin x - 2\sin x\cos x = 0$$

$$2\sin x\cos^2 x + 2\sin x - 2\sin^3 x - 2\sin x\cos x = 0$$

$$2\sin x\left[\cos^2 x + \underbrace{1-\sin^2 x}_{\cos^2 x} - \cos x\right] = 0$$

$$2\sin x\left[2\cos^2 x - \cos x\right] = 0$$

$$2\sin x\cos x\left[2\cos x - 1\right] = 0$$

$$2\sin x = 0, \quad \cos x = 0, \quad \cos x = \frac{1}{2}$$

$$\boxed{x = 0, \pi, \quad x = \frac{\pi}{2}, \frac{3\pi}{2}, \quad x = \frac{\pi}{3}, \frac{5\pi}{3}}$$