4. $\frac{-1}{\sqrt{2}}$

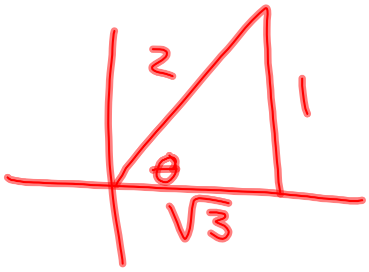
5. 1

6. $\frac{-1}{\sqrt{3}}$

7. $\frac{\pi}{6}$

8. $\frac{3\pi}{4}$

9. $-\frac{\pi}{2}$



$$LHS = \frac{\cos^2 x - \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \cot^2 x - 1$$

$$= RHS$$

75. $\tan\frac{x}{2} = \sin x$ $\qquad x \in [0, 2\pi)$

$$\frac{1-\cos x}{\sin x} = \sin x$$

$$1 - \cos x = \sin^2 x$$

$$1 - \cos x = 1 - \cos^2 x$$

$$\cos^2 x - \cos x = 0$$

$$\cos x(\cos x - 1) = 0$$

$$\cos x = 0 \qquad \cos x = 1$$

$$x = \pi/2,\ 3\pi/2,\ 0$$

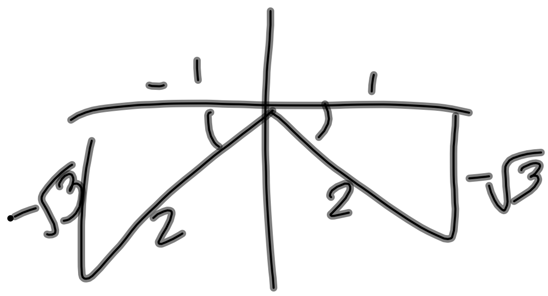
79. $\sin x\cos 2x - \cos x\sin 2x = \frac{\sqrt{3}}{2}$

$$\sin(x - 2x) = \frac{\sqrt{3}}{2}$$

$$\sin(-x) = \frac{\sqrt{3}}{2}$$

$$-\sin x = \frac{\sqrt{3}}{2}$$

$$\sin x = -\frac{\sqrt{3}}{2}$$



$$\boxed{x = 4\pi/3,\ 5\pi/3}$$

77.

$\sin 3x = 0$

$3x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi$

$x = 0, \frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}$

73. $\underbrace{\sin 4x} - \sin 2x = 0$

$2\sin 2x \cos 2x - \sin 2x = 0$

$\sin 2x \left(2\cos 2x - 1\right) = 0$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\frac{7\pi}{12} = \frac{x}{2}$$

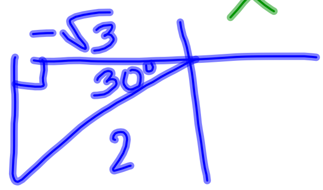
$x = ?$

$$\tan\frac{7\pi}{12} = \tan\frac{7\pi/6}{2}$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{-\frac{1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)(-2)$$

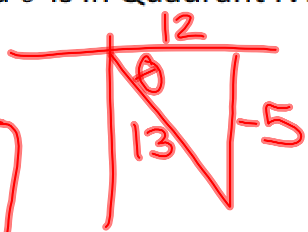
$$= \boxed{-2-\sqrt{3}}$$



2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$$= \cos(212° - 122°) = \cos 90° = \boxed{0}$$

3. Find $\sin 2\theta$, $\cos 2\theta$, *and* $\tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.



$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120/169}{119/169} = \boxed{\frac{-120}{119}}$$

What quadrant is 2θ in? IV

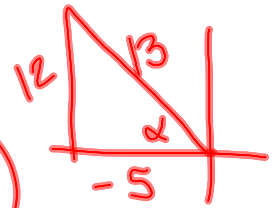
4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha+\beta)$.

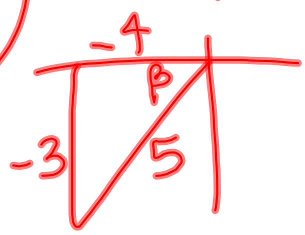
$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$= \left(\frac{12}{13}\right)\left(-\frac{4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$

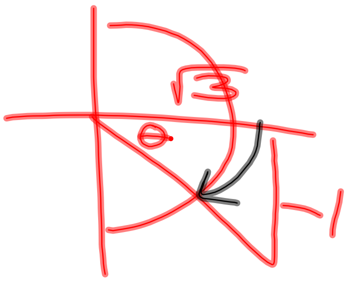
$$= \frac{-48}{65} + \frac{15}{65}$$

$$= \boxed{\frac{-33}{65}}$$



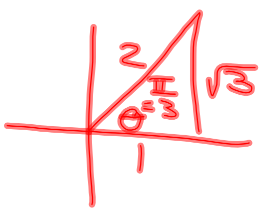


5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.

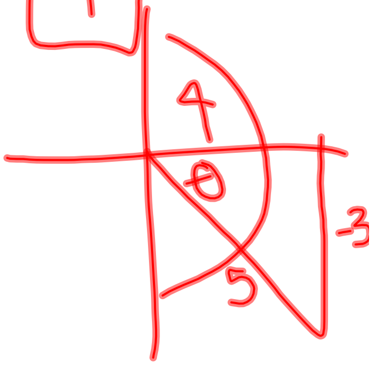


$$\boxed{\frac{-\pi}{6}}$$

6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right) = \cos\frac{\pi}{3} = \boxed{\frac{1}{2}}$



$$\sec\Big(\underbrace{\sin^{-1}\left(-\tfrac{3}{5}\right)}_{\theta}\Big) = \boxed{\frac{5}{4}}$$



7. Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm\frac{1}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

8. Find all solutions (in radians) in the interval $0 \leq x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x\,(2\sin^2 x - 1) = 0$

$\sin x = 0 \qquad \sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$x = 0, \pi, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$\text{LHS} = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x}$

$= \csc^2 x + \cot^2 x$

$= \csc^2 x + (\csc^2 x - 1)$

$= 2\csc^2 x - 1$

$= \text{RHS}$

$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1-\cos^2 x}{\sin x}$$

$$= \frac{\sin^{\cancel{2}} x}{\cancel{\sin x}}$$

$$= \sin x$$

$$= RHS$$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$

$$\sin(2x+x) + \sin x - 2\sin x\cos x = 0$$

$$\underline{\sin 2x}\cos x + \underline{\cos 2x}\sin x + \sin x - 2\sin x\cos x = 0$$

$$2\sin x\cos x \cdot \cos x + (2\cos^2 x - 1)\sin x + \sin x - 2\sin x\cos x = 0$$

$$2\sin x\cos^2 x + 2\sin x\cos^2 x - \cancel{\sin x} + \cancel{\sin x} - 2\sin x\cos x = 0$$

$$4\sin x\cos^2 x - 2\sin x\cos x = 0$$

$$2\sin x\cos x\,(2\cos x - 1) = 0$$

$2\sin x = 0$ $\quad \cos x = 0 \quad$ $2\cos x - 1 = 0$

$\sin x = 0 \qquad\qquad\qquad \cos x = \frac{1}{2}$

$$\boxed{x = 0,\ \pi,\ \frac{\pi}{2},\ \frac{3\pi}{2},\ \frac{\pi}{3},\ \frac{5\pi}{3}}$$