

Prove.

$$\frac{1}{1 + \cos x} - \frac{1}{1 - \cos x} = -2 \cot x \csc x$$

$$\begin{aligned} \text{LHS} &= \frac{1}{1 + \cos x} \cdot \frac{1 - \cos x}{1 - \cos x} - \frac{1}{1 - \cos x} \cdot \frac{1 + \cos x}{1 + \cos x} \quad \underline{\text{algebra}} \\ &= \frac{(1 - \cos x) - (1 + \cos x)}{1 - \cos^2 x} \quad \underline{\text{algebra}} \quad \frac{-2 \cos x}{\sin^2 x} \quad \underline{\text{algebra}} \\ &= \frac{-2 \cdot \cos x}{1} \cdot \frac{1}{\sin x} \quad \underline{\text{reciprocal}} \quad -2 \cot x \csc x = \text{RHS} \checkmark \\ &\quad \underline{\text{\& ratio id's}} \end{aligned}$$

Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\cos 3x + \frac{\sqrt{3}}{2} = 0$$

$$\cos 3x = -\frac{\sqrt{3}}{2}$$

$$2\pi \cdot \frac{6}{6} = \frac{12\pi}{6}$$

$$3x = \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{17\pi}{6}, \frac{19\pi}{6}, \frac{29\pi}{6}, \frac{31\pi}{6}$$

$$x = \frac{5\pi}{18}, \frac{7\pi}{18}, \frac{17\pi}{18}, \frac{19\pi}{18}, \frac{29\pi}{18}, \frac{31\pi}{18}$$

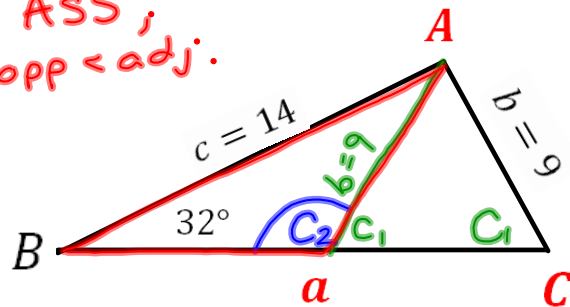
7.1 The Law of Sines, connued

ASS – Problematic Triangle

$$14. B = 32^\circ, c = 14, b = 9$$

Case 1: $C_1 \approx 55.5^\circ, A_1 \approx 92.5^\circ, a_1 \approx 17$

ASS ;
opp < adj.



Case 2:

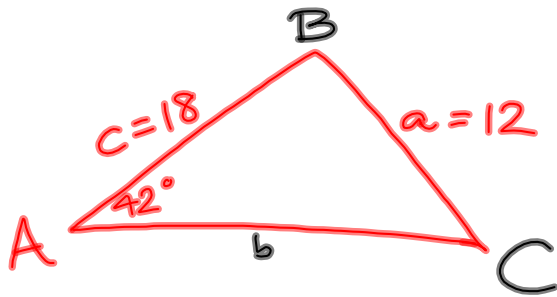
$$C_2 = 180^\circ - C_1 = 180^\circ - 55.5^\circ = 124.5^\circ$$

$$A_2 = 180^\circ - B - C_2 = 180^\circ - 32^\circ - 124.5^\circ = 23.5^\circ$$

$$\frac{a_2}{\sin 23.5^\circ} = \frac{9}{\sin 32^\circ}$$

$$a_2 = \frac{9 \sin 23.5^\circ}{\sin 32^\circ} \approx 6.8$$

16. $A = 42^\circ, a = 12, c = 18$



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$$\frac{\sin C}{18} = \frac{\sin 42^\circ}{12}$$

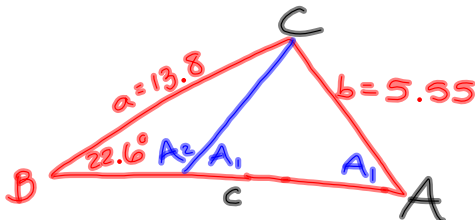
$$\sin C = \frac{18 \sin 42^\circ}{12}$$

$$C = \sin^{-1}\left(\frac{18 \sin 42^\circ}{12}\right) \text{ is undefined}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{18 \sin 42^\circ}{12}\right)$$



18. $B = 22.6^\circ, b = 5.55, a = 13.8$



ASS

$$\frac{\sin A}{13.8} = \frac{\sin 22.6^\circ}{5.55}$$

$$A = \sin^{-1}\left(\frac{13.8 \sin 22.6^\circ}{5.55}\right)$$

$$\approx 72.9^\circ$$

$$C = 180^\circ - 22.6^\circ - 72.9^\circ = 84.5^\circ$$

$$\frac{c}{\sin 84.5^\circ} = \frac{5.55}{\sin 22.6^\circ}$$

$$c = \frac{5.55 \sin 84.5^\circ}{\sin 22.6^\circ}$$

$$\approx 14.4$$

case 2

$$A = 180^\circ - 72.9^\circ = 107.1^\circ$$

$$C = 180^\circ - 22.6^\circ - 107.1^\circ = 50.3^\circ$$

$$\frac{c}{\sin 50.3^\circ} = \frac{5.55}{\sin 22.6^\circ}$$

$$c = \frac{5.55 \sin 50.3^\circ}{\sin 22.6^\circ} \approx 11.1$$

7.2 - The Law of Cosines

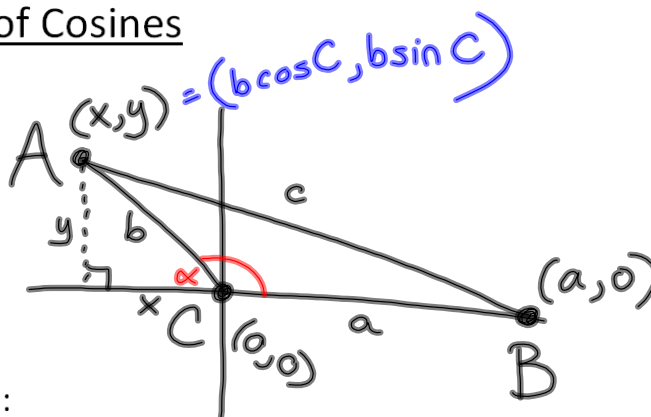
Derivation:

$$\cos C = \frac{x}{b}$$

$$x = b \cos C$$

$$\sin C = \frac{y}{b}$$

$$y = b \sin C$$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c^2 = (x-a)^2 + (y-0)^2$$

$$c^2 = (b \cos C - a)^2 + (b \sin C)^2$$

$$c^2 = b^2 \cos^2 C - 2ab \cos C + a^2 + b^2 \sin^2 C$$

$$c^2 = a^2 + b^2 (\underbrace{\sin^2 C + \cos^2 C}_{=1}) - 2ab \cos C$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cos C$$

7.2 Handout:

16. $a = 60, b = 88, c = 120. B = ?$

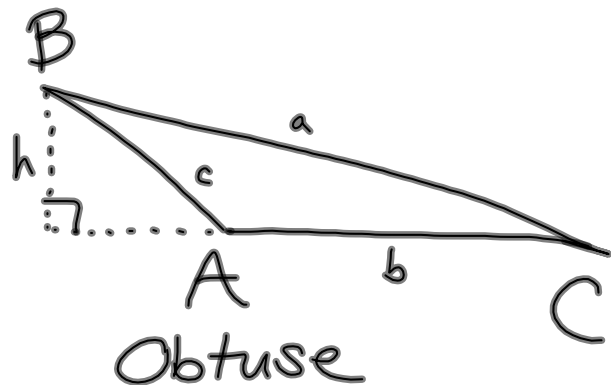
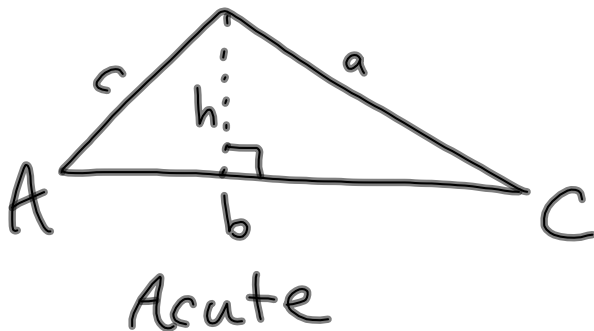
$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$2ac \cos B = a^2 + c^2 - b^2$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1} \left(\frac{a^2 + c^2 - b^2}{2ac} \right) = \cos^{-1} \left(\frac{60^2 + 120^2 - 88^2}{2(60)(120)} \right) \approx \boxed{44.6^\circ}$$

7.1/7.2 Area of a Triangle



Find the area of the triangle.

$$A = 50^\circ, b = 13 \text{ cm}, c = 6 \text{ cm}$$

Handout Homework:

7.1 #13-21 odd

7.2 #9-19 odd, (25-29 odd)