Prove.

$$\frac{1}{1+\cos x}-\frac{1}{1-\cos x}=-2\cot x\csc x$$

$\frac{a}{b}=\frac{c}{d}$ $ad=bc$

$LHS=\frac{1}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}-\frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x}$ $\overline{\overline{\text{algebra}}}$

$1-\cos x-1-\cos x$

$\frac{(1-\cos x)-(1+\cos x)}{1-\cos^2 x}$ $\overline{\overline{\text{algebra \& pythagorean id}}}$ $\frac{-2\cos x}{\sin^2 x}$ $\overline{\overline{\text{algebra}}}$

$\frac{-2}{1}\cdot\frac{\cos x}{\sin x}\cdot\frac{1}{\sin x}$ $\overline{\overline{\text{ratio \& reciprocal identities}}}$ $-2\cot x\csc x=RHS$

Find all solutions (in radians) in the interval $0\le x<2\pi$.

$\cos 3x+\frac{\sqrt{3}}{2}=0$

$\cos 3x=-\frac{\sqrt{3}}{2}$

$2\pi\cdot\frac{6}{6}=\frac{12\pi}{6}$

$3x=\frac{5\pi}{6},\frac{7\pi}{6},\frac{17\pi}{6},\frac{19\pi}{6},\frac{29\pi}{6},\frac{31\pi}{6}$

$$\boxed{x=\frac{5\pi}{18},\frac{7\pi}{18},\frac{17\pi}{18},\frac{19\pi}{18},\frac{29\pi}{18},\frac{31\pi}{18}}$$

## 7.1 The Law of Sines, connued

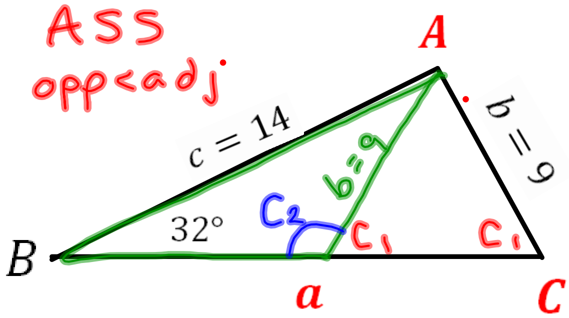
ASS

opp<adj

ASS – Problematic Triangle

14. $B=32°$, $c=14$, $b=9$

Case 1: $C_1\approx 55.5°$, $A_1\approx 92.5°$, $a_1\approx 17$




Case2

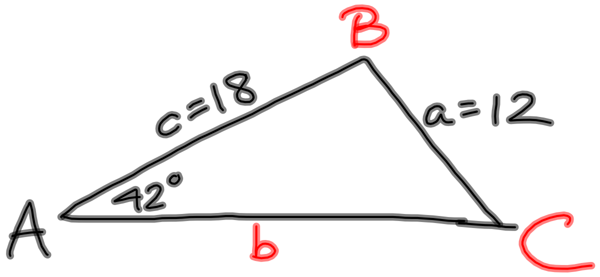
$C_2=180°-C_1=180°-55.5°=\boxed{124.5°}$

$A_2=180°-B-C_2=180°-32°-124.5°=\boxed{23.5°}$

$\frac{a_2}{\sin 23.5°}=\frac{9}{\sin 32°}$

$a_2=\frac{9\sin 23.5°}{\sin 32°}\approx\boxed{6.8}$

16. $A = 42°, a = 12, c = 18$




ASS

$a < c$

$$\frac{\sin C}{18} = \frac{\sin 42°}{12}$$

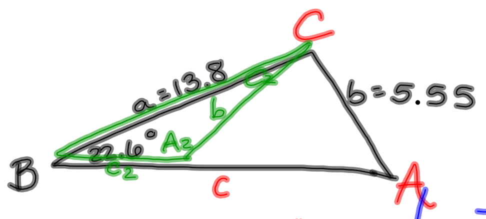
$$\sin C = \frac{18\sin 42°}{12}$$

$$\sin^{-1}(\sin C) = \sin^{-1}\left(\frac{18\sin 42°}{12}\right)$$

$$C = \sin^{-1}\left(\underbrace{\frac{18\sin 42°}{12}}_{>1}\right) \text{ is undefined}$$

$\boxed{\text{NO } \triangle \text{ exists}}$

18. $B = 22.6°, b = 5.55, a = 13.8$




ASS

$$\frac{\sin A}{13.8} = \frac{\sin 22.6°}{5.55}$$

$$A = \sin^{-1}\left(\frac{13.8\sin 22.6°}{5.55}\right)$$

$$\approx \boxed{72.9°}$$

$$C = 180° - 22.6° - 72.9°$$

$$= \boxed{84.5°}$$

$$\frac{c}{\sin 84.5°} = \frac{5.55}{\sin 22.6°}$$

$$c = \frac{5.55\sin 84.5°}{\sin 22.6°}$$

$$\approx \boxed{14.4}$$

Case 2

$$A_2 = 180° - 72.9°$$

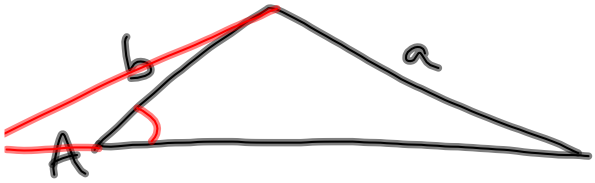
$$= \boxed{107.1°}$$

$$C_2 = 180° - 22.6° - 107.1°$$

$$= \boxed{50.3°}$$

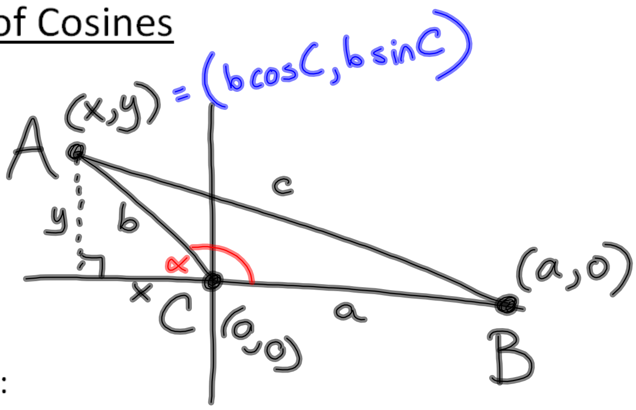
$$\frac{c_2}{\sin 50.3°} = \frac{5.55}{\sin 22.6°}$$

$$\boxed{c_2 = 11.1}$$




## 7.2 - The Law of Cosines

Derivation:




$\cos C = \frac{x}{b}$

$x = b\cos C$

$\sin C = \frac{y}{b}$

$y = b\sin C$

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c^2 = (x-a)^2 + (y-0)^2$$

$$c^2 = (b\cos C - a)^2 + (b\sin C)^2$$

$$c^2 = b^2\cos^2 C - 2ab\cos C + a^2 + b^2\sin^2 C$$

$$c^2 = a^2 + b^2(\underbrace{\sin^2 C + \cos^2 C}_{=1}) - 2ab\cos C$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

7.2 Handout:

16. $a = 60, b = 88, c = 120.\ B = ?$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

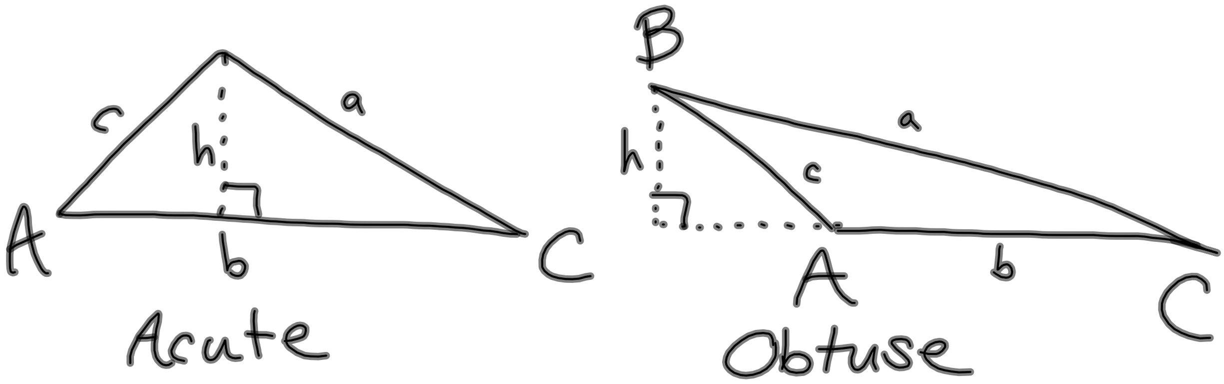
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1}\left(\frac{a^2 + c^2 - b^2}{2ac}\right) = \cos^{-1}\left(\frac{60^2 + 120^2 - 88^2}{2(60)(120)}\right)$$

$$\approx \boxed{44.6^\circ}$$

$$\cos^{-1}((60\text{^}2 + 120\text{^}2 - 88\text{^}2)/(2 * 60 * 120))$$

7.1/7.2 Area of a Triangle




Find the area of the triangle.

$A = 50°, b = 13\ cm, c = 6\ cm$

## Handout Homework:

7.1 #13-21 odd

7.2 #9-19 odd, (25-29 odd) area