Review: Angelica rides her magical bike at a rate of 120 miles per hour. The angular speed of each wheel is 528 revoluons per minute. What is the radius of a wheel, in inches?

$V = \frac{120 \text{ mi}}{h}$; $\omega = \frac{528 \text{ rev}}{\text{min}}$; $r = ? \text{ in}$

$\frac{V}{\omega} = \frac{r\cancel{\omega}}{\cancel{\omega}}$ $r = V \cdot \frac{1}{\omega}$

$r = \frac{\overset{2}{\cancel{120}} \cancel{\text{mi}}}{\cancel{h}} \cdot \frac{1 \cancel{\text{min}}}{\cancel{528} \cancel{\text{rev}}} \cdot \frac{\overset{10}{\cancel{5280}} \text{ ft}}{1 \cancel{\text{mi}}} \cdot \frac{12 \text{ in}}{1 \cancel{\text{ft}}} \cdot \frac{1 \cancel{h}}{\cancel{60} \cancel{\text{min}}} \cdot \frac{1 \cancel{\text{rev}}}{2\pi}$

$= \boxed{\frac{120}{\pi} \text{ in}}$

Homework questions?

Ch6 Test

23. $4\cos^2 x = 3$

$\cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\frac{\sin x \cos^2 x}{\sin x} = \frac{\sin x}{\sin x}$

$\cos^2 x = 1$

16. prove $\dfrac{1+\sin\alpha}{1+\csc\alpha} = \dfrac{\tan\alpha}{\sec\alpha}$ $\left(\csc\alpha = \frac{1}{\sin\alpha} = \frac{\sin\alpha+1}{\sin\alpha}\right)$

$\sin^2 x+\cos^2 x=1$
$1+\cot^2 x=\csc^2 x$
$1-\csc^2 x=-\cot^2 x$

$$LHS = \frac{1+\sin\alpha}{1+\csc\alpha}\cdot\frac{1-\csc\alpha}{1-\csc\alpha} = \frac{1-\csc\alpha+\sin\alpha-\sin\alpha\csc\alpha}{1-\csc^2\alpha}$$

$$= \frac{\cancel{1}-\frac{1}{\sin\alpha}+\sin\alpha-\cancel{\sin\alpha\frac{1}{\sin\alpha}}}{-\cot^2\alpha}$$

$$= \frac{-\frac{1}{\sin\alpha}+\sin\alpha}{\frac{-\cos^2\alpha}{\sin^2\alpha}} = \left(-\frac{1}{\sin\alpha}+\sin\alpha\right)\left(-\frac{\sin^2\alpha}{\cos^2\alpha}\right)$$

$$= \left(\frac{-1+\sin^2\alpha}{\sin\alpha}\right)\left(\frac{-\sin^2\alpha}{\cos^2\alpha}\right)$$

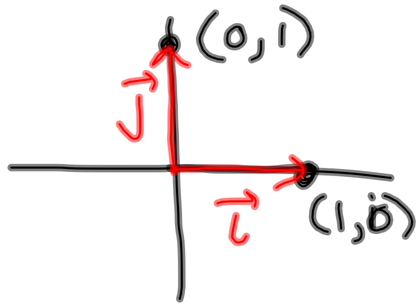
$$= \frac{+\cancel{\cos^2\alpha}}{\cancel{\sin\alpha}}\cdot\frac{+\sin^{\cancel{2}}\alpha}{\cancel{\cos^2}\alpha} = \sin\alpha\cdot\frac{\frac{1}{\cos\alpha}}{\frac{1}{\cos\alpha}}$$

$$= \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}} = \frac{\tan\alpha}{\sec\alpha} = RHS$$

$$RHS: \frac{\tan\alpha}{\sec\alpha} = \frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}} = \frac{\sin\alpha}{1}$$

7.6, cont. - Unit Vectors

A **unit vector** is a vector whose magnitude is 1.



Special Unit Vectors:

$\vec{\boldsymbol{i}} =< 1,0 >$ & $\vec{\boldsymbol{j}} =< 0,1 >$

In 3 dimensions we would have

$\vec{\boldsymbol{i}} =< 1,0,0 >$, $\vec{\boldsymbol{j}} =< 0,1,0 >$ & $\vec{\boldsymbol{k}} =< 0,0,1 >$

Any vector given in component form can also be written in terms of $\vec{\imath}$ and $\vec{\jmath}$.

$\vec{v} = \langle a, b \rangle$

$= \langle a, 0 \rangle + \langle 0, b \rangle$

$= a\langle 1, 0 \rangle + b\langle 0, 1 \rangle$

$\vec{v} = a\vec{\imath} + b\vec{\jmath}$

Vector operations for vectors given in terms of $\vec{\imath}$ and $\vec{\jmath}$ are in some ways simpler than for vectors given in component form, as $\vec{\imath}$ and $\vec{\jmath}$ can be treated like variables.

7.6 #46 & 48. $\vec{u} = 2\vec{\imath} + \vec{\jmath}$; $\vec{v} = -3\vec{\imath} - 10\vec{\jmath}$; $\vec{w} = \vec{\imath} - 5\vec{\jmath}$

46. $\vec{v} + 3\vec{w} = -3\vec{\imath} - 10\vec{\jmath} + 3(\vec{\imath} - 5\vec{\jmath})$

$= -3\vec{\imath} - 10\vec{\jmath} + 3\vec{\imath} - 15\vec{\jmath}$

$= \boxed{-25\vec{\jmath}} = \langle 0, -25 \rangle$

48. $(\vec{u} - \vec{v}) + \vec{w} =$ $2\vec{\imath} + \vec{\jmath} - (-3\vec{\imath} - 10\vec{\jmath}) + \vec{\imath} - 5\vec{\jmath}$

$= 2\vec{\imath} + \vec{\jmath} + 3\vec{\imath} + 10\vec{\jmath} + \vec{\imath} - 5\vec{\jmath}$

$= \boxed{6\vec{\imath} + 6\vec{\jmath}} = \langle 6, 6 \rangle$

Note that the magnitude of a vector given in the form $\vec{v} = a\vec{i} + b\vec{j}$ is still found by the formula $|\vec{v}| = \sqrt{a^2 + b^2}$

$a = 3 \quad b = -2$

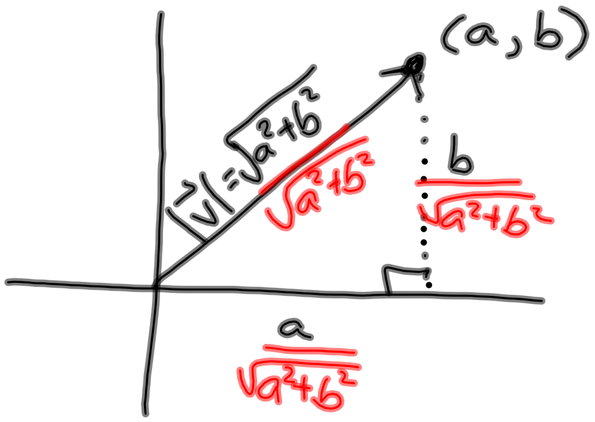
$\vec{v} = 3\vec{i} - 2\vec{j}$

$|\vec{v}| = \sqrt{3^2 + (-2)^2} = \sqrt{9+4} = \boxed{\sqrt{13}}$

$\cancel{= \sqrt{(3i)^2 + (-2j)^2}}$

$\cancel{\neq \sqrt{-1}}$

Given a vector $\vec{v} = < a, b >$, we can find a **unit vector $\vec{u}$ in the direction of $\vec{v}$** by dividing each component (a & b) by the magnitude $|\vec{v}|$.



$$\vec{u} = \frac{\vec{v}}{|\vec{v}|}$$

$$= \left\langle \frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}} \right\rangle$$

Given $\vec{v} = \langle -3, 4 \rangle$, find a unit vector $\vec{u}$ in the direction of $\vec{v}$.

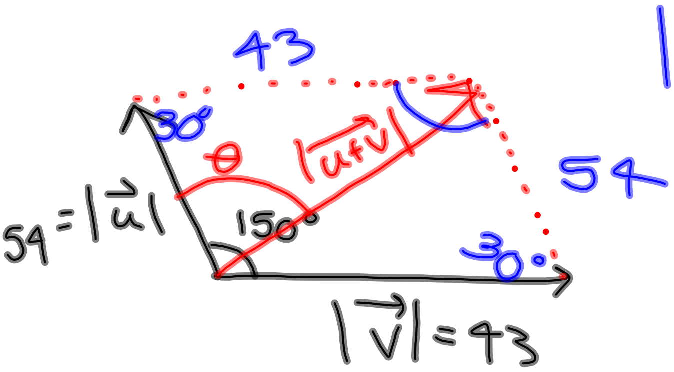
$$|\vec{v}| = \sqrt{a^2+b^2} = \sqrt{(-3)^2+4^2} = 5$$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \boxed{\left\langle \frac{-3}{5}, \frac{4}{5} \right\rangle}$$

$$|\vec{u}| = \sqrt{\left(\frac{-3}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{9}{25}+\frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1 \checkmark$$

Applications from section 7.5

18. Find $|\vec{u}+\vec{v}|$ and the angle that $\vec{u}+\vec{v}$ makes with $\vec{u}$, given $|\vec{u}| = 54$, $|\vec{v}| = 43$, & the angle θ between $\vec{u}$ & $\vec{v}$ is $150°$.



$$|\vec{u+v}| = \sqrt{43^2+54^2-2(43)(54)\cos 30°}$$

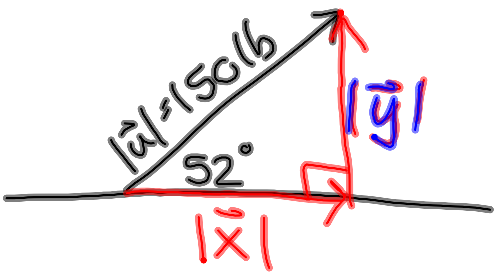
$$= \boxed{27.3}$$

$$\frac{\sin\theta}{43} = \frac{\sin 30°}{27.3}$$

$$\theta = \sin^{-1}\left(\frac{43\sin 30°}{27.3}\right) = \boxed{52°}$$

Resolving a vector into horizontal and vertical components

32. $|\vec{u}| = 150$ lb, inclined upward to the right at $52°$ from the horizontal. Resolve $\vec{u}$ into horizontal and vertical components.



Vertical component

$\sin 52° = \frac{|\vec{y}|}{150}$

$|\vec{y}| = 150\sin 52° = \boxed{118.2\text{lb}}$

horizontal component

$\cos 52° = \frac{|x|}{150}$

$|\vec{x}| = 150\cos 52° = \boxed{92.3\text{lb}}$

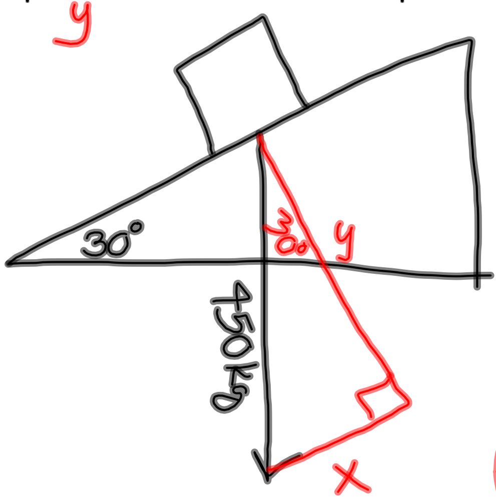
The object on a ramp problem

40. If a 450kg object is at rest on a ramp with a $30°$ incline, find the components of the force of the object's weight parallel and perpendicular to the ramp.

x

"Normal"

y



★ weight is always straight down

parallel:

$\sin 30° = \frac{|x|}{450}$

$|\vec{x}| = 450\sin 30° = \boxed{225\text{ kg}}$

perpendicular:

$\cos 30° = \frac{|\vec{y}|}{450}$

$|\vec{y}| = 450\cos 30° = \boxed{389.7\text{ kg}}$

Homework:

Already assigned: 7.5 #27,29; 7.6 #9-26 all

7.5# 19,21,33,39

7.6# 33-41odd, 45, 47, 57, 61

Due Friday