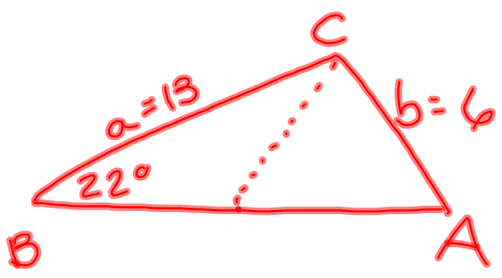
Commonly missed problems from Test #4:

5. Find **angle A**. $a = 13, b = 6, \; B = 22°$

$$\frac{x}{\sin 97°} = \frac{18}{\sin 78°}$$

$$\boxed{x = 18.3 \text{ mi}}$$

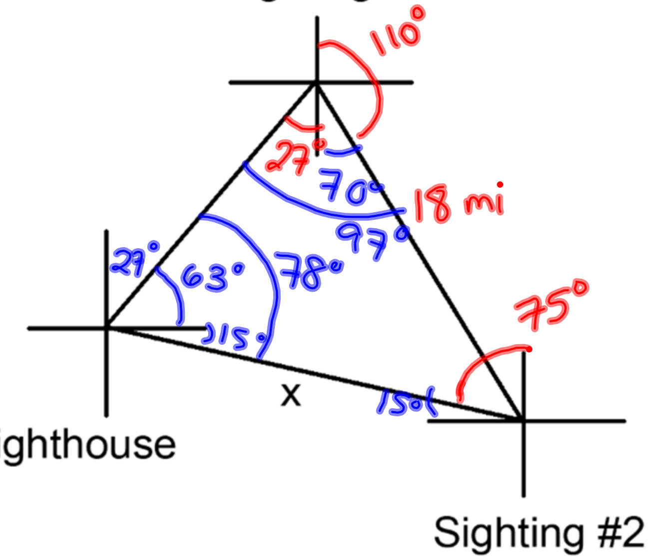


Case 1 A : 54.3°

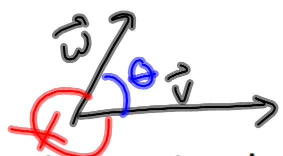
Case 2 : 180° − 54.3°

= 125.7°

9.



<u>7.6 Vectors</u>, cont.



The smallest nonnegative <u>angle between two vectors</u> is given by

$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$, or equivalently, $\boxed{\theta = \cos^{-1}\frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}}$.

Recall that both the dot product and magnitude vector operations yield a scalar (non-vector real number) quantity, so we can find the inverse cosine value.

Given $\vec{v} = \langle a, b\rangle$ and $\vec{w} = \langle c, d\rangle$,

$$|\vec{v}| = \sqrt{a^2 + b^2}$$

$$\vec{v}\cdot\vec{w} = ac + bd$$

7.6 #64. Given $\vec{a} = \langle -3, -3 \rangle$ and $\vec{b} = \langle -5, 2 \rangle$, find the smallest non-negative angle between $\vec{a}$ and $\vec{b}$.

$$\cos\theta = \frac{\vec{v_1} \cdot \vec{v_2}}{|\vec{v_1}||\vec{v_2}|}$$

$$\vec{a} \cdot \vec{b} = (-3)(-5) + (-3)(2) = 15 - 6 = \boxed{9}$$

$$|\vec{a}| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = \boxed{3\sqrt{2}}$$

$$|\vec{b}| = \sqrt{(-5)^2 + 2^2} = \boxed{\sqrt{29}}$$

$$\theta = \cos^{-1}\left(\frac{9}{3\sqrt{2} \cdot \sqrt{29}}\right) \approx \boxed{66.8^\circ}$$

68. $\vec{u} = 3\vec{i} + 2\vec{j}$; $\vec{v} = -\vec{i} + 4\vec{j}$

$$|\vec{u}| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

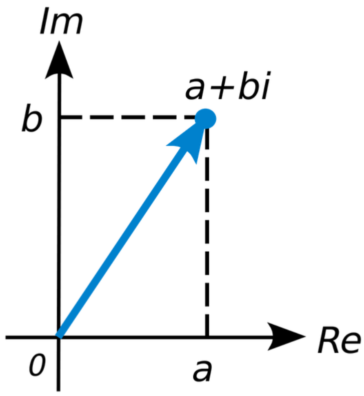
$$|\vec{v}| = \sqrt{(-1)^2 + 4^2} = \sqrt{17}$$

$$\vec{u} \cdot \vec{v} = 3(-1) + 2(4) = -3 + 8 = 5$$

$$\theta = \cos^{-1}\left(\frac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}\right) = \cos^{-1}\left(\frac{5}{\sqrt{13 \cdot 17}}\right)$$

$$\approx \boxed{70.3^\circ}$$

7.3 Trigonometric Form of Complex Numbers



Complex Number review:

$z = a + bi$, where $i = \sqrt{-1}$; $a, b \in \mathbb{R}$

a is the "real component"

b is the "imaginary component"

The modulus of a complex number is its distance from the origin.

$|z| = \sqrt{a^2 + b^2}$

The argument θ of a complex number is the direction angle, measured counter-clockwise from the positive x-axis.

Trigonometric Form of Complex Numbers

r = modulus

θ = argument

$a + bi$

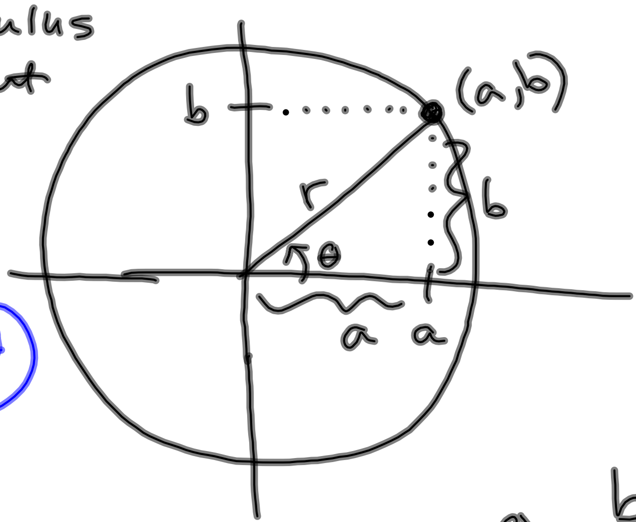
standard form

$= r\cos\theta + r\sin\theta \cdot i$

$= r(\cos\theta + i\sin\theta)$

$= r\,\text{cis}\,\theta$

trigonometric form



$\cos\theta = \frac{a}{r}$ $\sin\theta = \frac{b}{r}$

$a = r\cos\theta$ $b = r\sin\theta$

Multiplying complex #'s in trigonometric form

$z_1 = r_1 cis\theta_1$; $z_2 = r_2 cis\theta_2$

$z_1 z_2 = (r_1\cos\theta_1 + ir_1\sin\theta_1)(r_2\cos\theta_2 + ir_2\sin\theta_2)$

$= r_1r_2\cos\theta_1\cos\theta_2 + ir_1r_2\cos\theta_1\sin\theta_2 + ir_1r_2\sin\theta_1\cos\theta_2 + i^2 r_1r_2\sin\theta_1\sin\theta_2$

$i^2 = -1$

$= r_1r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) + ir_1r_2(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$

$= r_1r_2\cos(\theta_1+\theta_2) + ir_1r_2\sin(\theta_1+\theta_2)$

$= r_1r_2\, cis(\theta_1+\theta_2)$

To mulply two complex numbers in trigonometric form, mulply the moduli and add the arguments.

$z_1 = -2cis20°$; $z_2 = 5cis40°$

$z_1 z_2 = (-2)(5)\, cis(20° + 40°)$

$= \boxed{-10\, cis\, 60°}$

$z_1 z_2 \cdots z_n = r_1 r_2 \cdots r_n\, cis(\theta_1 + \cdots + \theta_n)$

$z^n = r^n cis(n\theta)$

Dividing complex #'s in trigonometric form

$z_1 = r_1 cis\theta_1$; $z_2 = r_2 cis\theta_2$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2)$$

To divide two complex numbers in trigonometric form, divide the moduli and subtract the arguments.

$z_1 = 12cis120°$; $z_2 = -4cis10°$

$$\frac{z_1}{z_2} = \frac{12}{-4} cis(120° - 10°) = \boxed{-3cis110°}$$

Converting between

Standard Form & Trigonometric Form

$a + bi$ $\quad$ $rcis\theta$

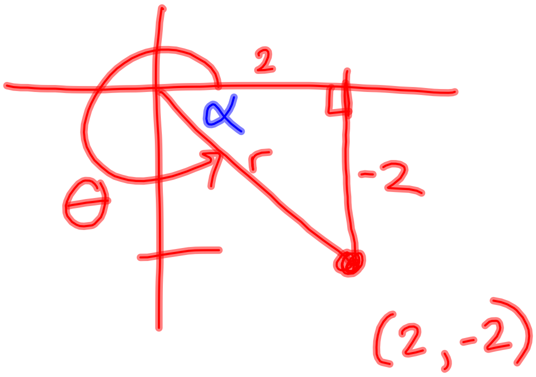
$r(\cos\theta + i\sin\theta)$

$$z = 5cis30° = 5(\cos 30° + i\sin 30°)$$

$$= 5\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right)$$

$$= \frac{5\sqrt{3}}{2} + i \cdot \frac{5}{2} = \boxed{\frac{5\sqrt{3}}{2} + \frac{5}{2}i}$$

$z = 2 - 2i =$



$r = \sqrt{2^2 + (-2)^2}$

$= \sqrt{8} = 2\sqrt{2}$

$\alpha = 45^\circ \Rightarrow \theta = 315^\circ$

$z = \boxed{2\sqrt{2} \operatorname{cis} 315^\circ}$

<u>Homework</u>:

<u>7.6</u> #63,65,67 – determine angle between vectors

<u>7.3</u> #13-43 odd – trig form of complex #'s

Final Exam Pracce Problems