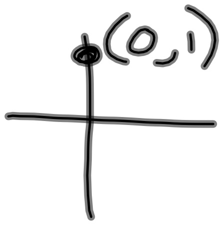
Cofunction Identities

$$\cos\left(\frac{\pi}{2}-x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$

$$= 0 \cdot \cos x + 1 \cdot \sin x$$

$$= \sin x$$



Double-Angle Identities

$\sin(2x) = \sin(x+x)$

$= \sin x \cos x + \cos x \sin x$

$\boxed{\sin 2x = 2\sin x \cos x}$

$\sin 6x = \sin 2(3x) = 2\sin 3x \cos 3x$

$\sin 8x = \sin 2(4x) = 2\sin 4x \cos 4x$

$\cos 2x = \cos(x+x) =$

$= \cos x \cos x - \sin x \sin x$

$\boxed{\cos 2x = \cos^2 x - \sin^2 x}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\cos^2 x = 1 - \sin^2 x$

$= (1-\sin^2 x) - \sin^2 x$

$\boxed{\cos 2x = 1 - 2\sin^2 x}$

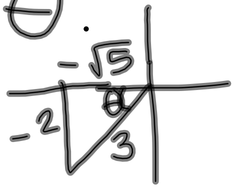
$= \cos^2 x - (1-\cos^2 x)$

$\boxed{\cos 2x = 2\cos^2 x - 1}$

$$\tan 2x = \tan(x+x) =$$
$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$

$$\boxed{\tan 2x = \frac{2\tan x}{1-\tan^2 x}}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in Q\,\text{III}$,
Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$.



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\cdot\left(\frac{-2}{3}\right)\cdot\left(\frac{-\sqrt{5}}{3}\right)$$
$$= \frac{4\sqrt{5}}{9}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$$
$$= \frac{5}{9} - \frac{4}{9}$$
$$= \boxed{\frac{1}{9}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4\sqrt{5}}{9}}{1/9}$$
$$= \frac{4\sqrt{5}}{\cancel{9}} \cdot \frac{\cancel{9}}{1} = \boxed{4\sqrt{5}}$$

2θ is in Quadrant I

Half-Angle Identities

$\sin\left(\frac{x}{2}\right) = ?$

$\cos 2x = 2\cos^2 x - 1$, $\cos 2x = 1 - 2\sin^2 x$

$\cos 2x + 1 = 2\cos^2 x$ $2\sin^2 x = 1 - \cos 2x$

$\frac{\cos 2x + 1}{2} = \cos^2 x$ $\sin^2 x = \frac{1 - \cos 2x}{2}$

$\pm\sqrt{\frac{\cos 2x + 1}{2}} = \cos x$ $\sin x = \pm\sqrt{\frac{1 - \cos 2x}{2}}$

Let $x = \frac{\theta}{2}$ Let $x = \frac{\theta}{2}$

$\pm\sqrt{\frac{\cos\theta + 1}{2}} = \cos\frac{\theta}{2}$ $\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}$

$$\boxed{\tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}}$$

$$= \frac{\sin\theta}{1 + \cos\theta}$$

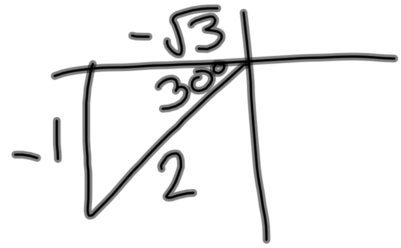
$$= \frac{1 - \cos\theta}{\sin\theta}$$

$$\tan\frac{7\pi}{12} = \tan\left(\frac{\frac{7\pi}{6}}{2}\right) =$$

$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} =$$

$$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\cdot\left(\frac{-2}{1}\right)$$



$$= \boxed{-2-\sqrt{3}}$$

☆Memorize Identities!!!!

Quiz Mon or Tues

6.3 handout:

#1-24, 30-36