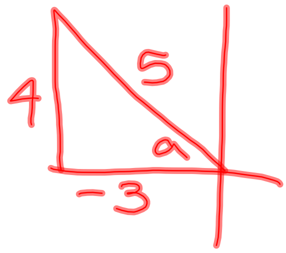
Review:

Given $\csc a = \frac{5}{4}$, $a \in QII$, and $\tan b = \frac{1}{2}$, $b \in QIII$, find:

$\cos(a-b) =$ cosacosb + sinasinb

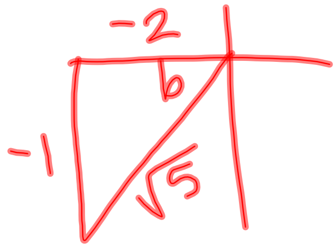
$= \left(\frac{-3}{5}\right)\left(\frac{-2}{\sqrt{5}}\right) + \left(\frac{4}{5}\right)\left(\frac{-1}{\sqrt{5}}\right)$

$= \boxed{\frac{2}{5\sqrt{5}}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \boxed{\frac{2\sqrt{5}}{25}}$



$\sin(2a) =$ 2sinacosa

$= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right) = \boxed{\frac{-24}{25}}$



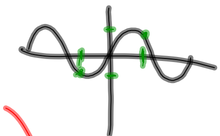
$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a}$

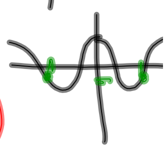
$= \frac{5}{5} \cdot \frac{1-\left(\frac{-3}{5}\right)}{\frac{4}{5}} = \frac{\overset{2}{\cancel{8}}}{\cancel{5}} \cdot \frac{\cancel{5}}{\cancel{4}} = \boxed{2}$

Odd-Even Identities

$\cos(-x) = \cos x$, $\sin(-x) = -\sin x$, $\tan(-x) = -\tan x$
$\sec(-x) = \sec x$, $\csc(-x) = -\csc x$, $\cot(-x) = -\cot x$

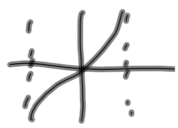
A function f is
even if $f(-x) = f(x)$
odd if $f(-x) = -f(x)$





$\cos(-x) = \cos(0-x) =$
$= \cos 0 \cos x + \sin 0 \sin x$
$= 1 \cdot \cos x + 0 \cdot \sin x$
$= \cos x$

$\tan(-x) = \frac{\sin(-x)}{\cos(-x)} = \frac{-\sin x}{\cos x} = -\tan$



Tues:

non-proof identity problens (6.2 # 1–42, 6.3 # 1–48)

Friday

proofs

turn in 6.3 handout 49–93 odd

6.3 handout Prove.

50. $\cos 8x = \cos^2 4x - \sin^2 4x$

$\cos(4x+4x)$

$LHS = \cos 2(4x) =$

$= \cos^2(4x) - \sin^2(4x) =$

$= RHS$ ✓

52. $\dfrac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

$$LHS = \frac{\cos^2 x - \sin^2 x}{\sin^2 x}$$

$$= \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x}$$

$$= \cot^2 x - 1$$

$$= RHS \checkmark$$

$$\boxed{\begin{array}{l} \cos 2x = \\ \cos^2 x - \sin^2 x \\ 1 - 2\sin^2 x \\ 2\cos^2 x - 1 \end{array}}$$

54. $\dfrac{1}{1-\cos 2x} = \dfrac{1}{2}\csc^2 x$

$$LHS = \frac{1}{1-(1-2\sin^2 x)} = \frac{1}{2\sin^2 x}$$

$$= \frac{1}{2}\csc^2 x = RHS \checkmark$$

56. $$\frac{\cos^2 x - \sin^2 x}{2\sin x \cos x} = \cot 2x$$

$$LHS = \frac{\cos 2x}{\sin 2x} = \cot 2x = RHS$$

60. $$\sin 2x - \cot x = -\cot x \cos 2x$$

$$RHS = -\cot x\left(1 - 2\sin^2 x\right)$$
$$= -\cot x + 2\sin^2 x \cot x$$
$$= -\cot x + 2\sin x \cancel{\sin x} \cdot \frac{\cos x}{\cancel{\sin x}}$$
$$= -\cot x + 2\sin x \cos x$$
$$= -\cot x + \sin 2x$$
$$= LHS \checkmark$$

62. $\sin 4x = 4\sin x\cos^3 x - 4\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x$

$= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x)$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x$

$= RHS$

64. $2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x = \cos^2 2x$

$RHS = (\cos 2x)^2 = \cos 2x\cos 2x$

$= (\cos^2 x - \sin^2 x)(2\cos^2 x - 1)$

$= 2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x$

$= LHS$

66. $\sin 4x = 4\sin x\cos x - 8\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x$

$= 2(2\sin x\cos x)(1-2\sin^2 x)$

$= 4\sin x\cos x - 8\sin^3 x\cos x$

$= RHS$

68. $\sin 3x + \sin x = 4\sin x - 4\sin^3 x$

$LHS = \sin(2x+x) + \sin x$

$\boxed{\begin{array}{l}\sin^2 x + \cos^2 x = \\ \cos^2 x = 1-\sin\end{array}}$

$= \sin 2x\cos x + \cos 2x\sin x + \sin x$

$= (2\sin x\cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x + \sin x$

$= 2\sin x\cos^2 x + \sin x\cos^2 x - \sin^3 x + \sin x$

$= 3\sin x(\cos^2 x) - \sin^3 x + \sin x$

$= 3\sin x(1-\sin^2 x) - \sin^3 x + \sin x$

$= 3\sin x - 3\sin^3 x - \sin^3 x + \sin x$

$= 4\sin x - 4\sin^3 x$

$= RHS$

#49-67
6.3 handout
proofs