6.3 handout problems – prove the trigonometric identy $a^3-b^3=(a-b)(a^2+ab+b^2)$

70. $\cos^3 x - \sin^3 x = (\cos x - \sin x)\left(1 + \frac{1}{2}\sin 2x\right)$

$LHS = (\cos x - \sin x)(\cos^2 x + \sin x\cos x + \sin^2 x) =$

$= (\cos x - \sin x)(1 + \sin x\cos x)$ ($\cos^2 x + \sin^2 x = 1$)

$= (\cos x - \sin x)\left(1 + \frac{1}{2}\cdot 2\sin x\cos x\right)$

$= (\cos x - \sin x)(1 + \frac{1}{2}\sin 2x)$

$= RHS$

72. $\cos^2 \frac{x}{2} = \frac{\sec x + 1}{2\sec x}$

$LHS = \left(\cos\frac{x}{2}\right)^2 = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \frac{1+\cos x}{2}$

$= \frac{(1+\cos x)}{2}\cdot\frac{\sec x}{\sec x} = \frac{\sec x + \cos x\sec x}{2\sec x} =$

$= \frac{\sec x + \cos x\cdot\frac{1}{\cos x}}{2\sec x} = \frac{\sec x + 1}{2\sec x} = RHS$

74. $\tan\frac{x}{2} = \frac{\tan x}{\sec x + 1}$

$$LHS = \tan\frac{x}{2} = \frac{\sin x}{(1+\cos x)} \cdot \frac{\frac{1}{\cos x}}{\frac{1}{\cos x}}$$

$$= \frac{\frac{\sin x}{\cos x}}{\frac{1}{\cos x} + \cos x \cdot \frac{1}{\cos x}} = \frac{\tan x}{\sec x + 1} = RHS$$

76. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \cos x$

$$LHS = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 - \left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 =$$

$$= \frac{1+\cos x}{2} - \frac{(1-\cos x)}{2} = \frac{1+\cos x - 1 + \cos x}{2}$$

$$= \frac{2\cos x}{2} = \cos x = RHS$$

80. $\cos^2\frac{x}{2}\sec x = \frac{1}{2}(\sec x + 1)$

$$LHS = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 \cdot \sec x =$$

$$= \frac{1+\cos x}{2} \cdot \sec x =$$

$$= \frac{\sec x + \cos x \sec x}{2} = \frac{1}{2}\left(\sec x + \cos x \cdot \frac{1}{\cos x}\right)$$

$$= \frac{1}{2}(\sec x + 1) = RHS$$

82. $\sin^2\frac{x}{2} + \cos x = \cos^2\frac{x}{2}$

$$LHS = \left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2 + \cos x =$$

$$= \frac{1-\cos x}{2} + \frac{\cos x}{1} \cdot \frac{2}{2}$$

$$= \frac{1-\cos x + 2\cos x}{2}$$

$$= \frac{1+\cos x}{2}$$

$$= \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 = \cos^2\frac{x}{2} = RHS$$

84. $\cos^2\frac{x}{2} - \sin^2\frac{x}{2} = \frac{1}{2}\csc x \sin 2x$

$$LHS = \left(\pm\sqrt{\frac{1+\cos x}{2}}\right)^2 - \left(\pm\sqrt{\frac{1-\cos x}{2}}\right)^2$$

$$= \frac{1+\cos x}{2} - \frac{1-\cos x}{2}$$

$$= \frac{1+\cos x - 1 + \cos x}{2}$$

$$= \frac{2\cos x}{2}$$

$$= \cos x \cdot \frac{\sin x}{\sin x} \cdot \frac{2}{2}$$

$$= \frac{2\sin x \cos x}{\sin x} \cdot \frac{1}{2} \cdot \frac{1}{\sin x}$$

$$= \sin 2x \cdot \frac{1}{2} \cdot \csc x$$

$$= RHS$$

Homework: 6.3 handout #69-83 odd