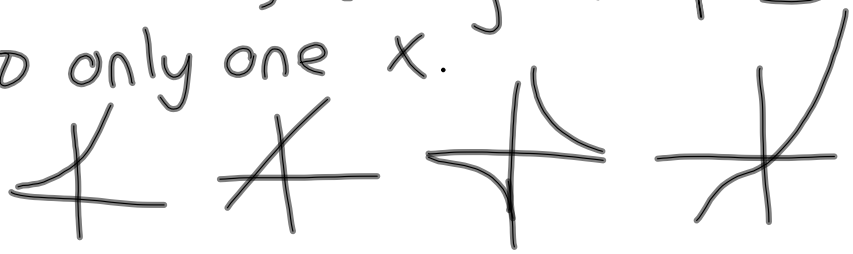
Inverse Trig Functions

6.4 book / 6.5 handout

- f is a function if each input value (x) has a unique output f(x).
- f is one-to-one if, in addition, each y corresponds to only one x.



- if we have a one to one function f we can define its inverse f^{-1}

$f(x)$ & $f^{-1}(x)$ are inverses if $(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$.

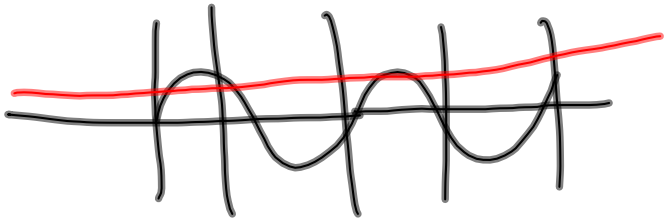
$(f \circ g)(x) = f(g(x))$

Inverse functions "undo" each other.

$f(x) = x^3$ $\quad g(x) = \sqrt[3]{x}$

$f(g(x)) = \left(\sqrt[3]{x}\right)^3 = x$

$g(f(x)) = \sqrt[3]{x^3} = x$



To define inverses of Trig functions we must restrict the domain.

$f(x) = \sin x$

$y = \sin x$

$x = \sin y$

x (input) = angle

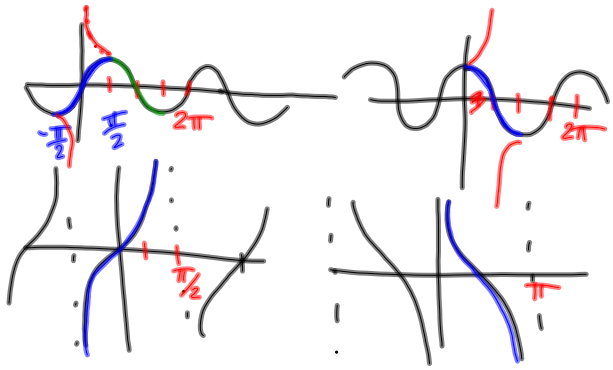
$f(x)$ (output) = ratio of sides

y = the angle whose sine value is x

$y = \sin^{-1} x$ or $y = \arcsin x$

$\sin^{-1} x \neq \frac{1}{\sin x}$

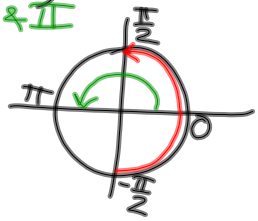
$f^{-1}(x) = \sin^{-1} x$ $\quad f^{-1}(x) = \arcsin x$



Restricted Domains:

$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$: sin, csc, & tan
Q's IV & I

$(0, \pi)$: cos, sec, & cot
Q's I & II



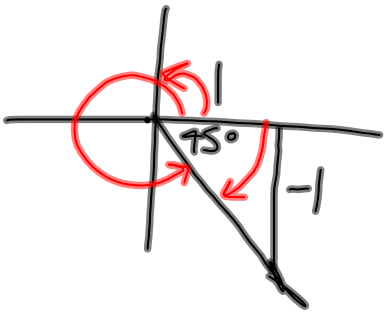
$\sin^{-1}\left(\frac{1}{2}\right) = ?$ 30° or $\pi/6$

What angle between $\frac{-\pi}{2}$ & $\frac{\pi}{2}$ has a sine value of $\frac{1}{2}$?

$\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$ or 120°

What angle between 0 & π has a cosine value of $-\frac{1}{2}$?

$\tan^{-1}(-1) = \frac{-\pi}{4}$ or $-45°$

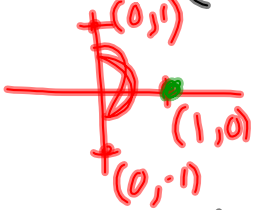


is $\frac{7\pi}{4}$ between $\frac{-\pi}{2}$ & $\frac{\pi}{2}$?

$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = 120° = \frac{2\pi}{3}$

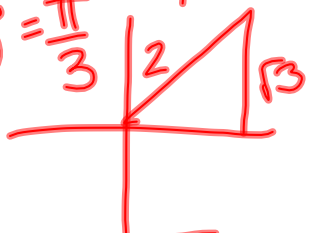
(0, π)

$\tan^{-1}(0) = 0° = ($



$\sin^{-1}\frac{\sqrt{3}}{2} = 60° = \frac{\pi}{3}$

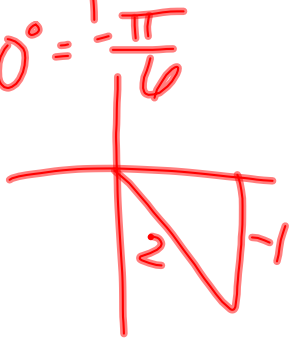
$\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$



$\cos^{-1}(3)$ undefined

$\csc^{-1}(-2) = -30° = \frac{-\pi}{6}$

$\frac{-\pi}{2}, \frac{\pi}{2}$



f & g are inverses if

$f(g(x)) = x$ & $g(f(x)) = x$

$\forall x$ in the domain of $\forall x$ in the domain of f

$\sin(\sin^{-1}x) = x$ & $\sin^{-1}(\sin x) = x$

$\sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{2}$

$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = -\frac{\pi}{6}$

$\sin(\sin^{-1} 3)$ undefined

6.5 handout #1-24

$\sin^{-1}\left(\sin\frac{5\pi}{6}\right) =$

$\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\cos^{-1}\left(\cos\frac{8\pi}{7}\right) = \frac{6\pi}{7}$