6.5 handout

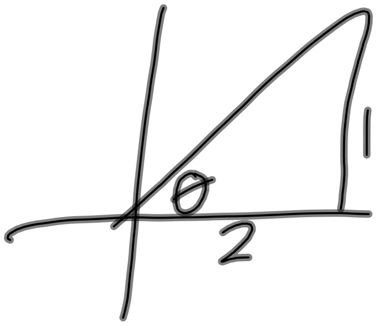
35. $\sin^{-1}\left(\tan\frac{\pi}{3}\right)$

$= \sin^{-1}\left(\sqrt{3}\right)$ undefined

$\left(\sqrt{3} > 1\right)$

37. $\tan^{-1}\left(\sin\frac{\pi}{6}\right)$

$\tan^{-1}\left(\frac{1}{2}\right)$



49. $\cos\left(2\underbrace{\sin^{-1}\frac{1}{\sqrt{2}}}_{\theta}\right)$

$= \cos\left(2 \cdot 45^\circ\right)$

$= \cos 90^\circ$

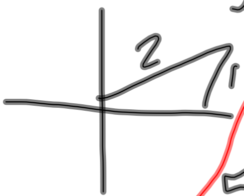
$= \boxed{0}$

Solving Trigonometric Equations!

$\sin^{-1}\left(\frac{1}{2}\right)$ v. $\sin x = \frac{1}{2}$

has 1 answer $\frac{\pi}{6}$

has infinitely many answers



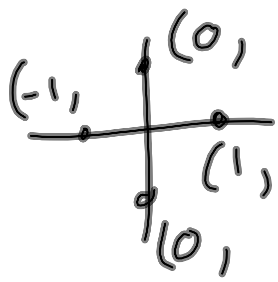
$\frac{\pi}{6} + 2\pi k$

$\frac{5\pi}{6} + 2\pi k$

$k \in \mathbb{Z}$ is any integer

<u>6.4 handout</u>

Solve for $x \in [0, 2\pi)$



2. $2\sin x = \sqrt{3}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

4. $\cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = 0}$

6. $2\sin x\cos x = \sqrt{3}\sin x$

Algebraic Interlude . . .

$(x-2)(x-3)(x-4)=0$

$\Rightarrow x-2=0,\ x-3=0,\ x-4=0$

$x=2,3,4$

Zero Product Property:

If $AB=0$, then

$A=0$ or $B=0$.

$(x-1)(x-3)=4$

$x^2=9$

$x=\pm 3$

Square Root Theorem

If $[f(x)]^2=C$, then

$f(x)=\pm\sqrt{C}$

6. $2\sin x\cos x = \sqrt{3}\sin x$

$x^3 - x^2 = 0$

$x^2(x-1) = 0$

$x^2 = 0 \quad x-1=0$

$x = 0 \quad x = 1$

$\frac{x^3}{x^2} = \frac{x^2}{x^2}$

$x = 1$

$2\sin x\cos x = \sqrt{3}\sin x$

$\frac{x^3}{x} = \frac{x^2}{x}$

$2\sin x\cos x - \sqrt{3}\sin x = 0$

$x^2 = x$

$\sin x(2\cos x - \sqrt{3}) = 0$

$\sin x = 0 \qquad 2\cos x - \sqrt{3} = 0$

$x = 0, \pi$

$2\cos x = \sqrt{3}$

$\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{11\pi}{6}$

8. $\cos^2 x - 1 = 0$

$(\cos x)^2 - 1^2 = 0$

$\cos^2 x = 1$

$(\cos x + 1)(\cos x - 1) = 0$

$\cos x = \pm 1$

$\cos x + 1 = 0, \cos x - 1 = 0$

$\cos x = -1, \cos x = 1$

$\boxed{x = 0, \pi}$

10. $\underbrace{\sec^2 x + \sqrt{3}\sec x} - \underbrace{\sqrt{2}\sec x - \sqrt{6}} = 0$

$\sec x(\sec x + \sqrt{3}) - \sqrt{2}(\sec x + \sqrt{3}) = 0$

$(\sec x + \sqrt{3})(\sec x - \sqrt{2}) = 0$

$\sec x + \sqrt{3} = 0$ $\quad$ $\sec x - \sqrt{2} = 0$

$\sec x = -\sqrt{3}$ $\quad$ $\sec x = \sqrt{2}$

$x =$ 2 not-so-nice angles

$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$

14. $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

Let $u = \cos x$

$2u^2 + 3u + 1 = 0$

$(2u+1)(u+1) = 0$

$(2\cos x + 1)(\cos x + 1) = 0$

$\cos x = -\frac{1}{2}$ $\quad$ $\cos x = -1$

$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3}}$ $\quad$ $\boxed{x = \pi}$

18. $4\cos^3 x = 3\cos x$

$4\cos^3 x - 3\cos x = 0$

$\cos x\,(4\cos^2 x - 3) = 0$

$\cos x = 0 \qquad 4\cos^2 x - 3 = 0$

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad \cos^2 x = \frac{3}{4}$

$\cos x = \pm\frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3}\tan x$

$\underbrace{\tan^2 x + \tan x} - \underbrace{\sqrt{3}\tan x - \sqrt{3}} = 0$

$\tan x\,(\tan x + 1) - \sqrt{3}\,(\tan x + 1) = 0$

$(\tan x + 1)(\tan x - \sqrt{3}) = 0$

$\tan x = -1 \qquad \tan x = \sqrt{3}$

$x = 3\pi/4,\ 7\pi/4 \qquad x = \frac{\pi}{3}, \frac{4\pi}{3}$

22. $\cos^4 x = \cos^2 x$

$\cos^4 x - \cos^2 x = 0$

$\cos^2 x(\cos^2 x - 1) = 0$

$\cos^2 x = 0 \qquad \cos^2 x - 1 = 0$

$\cos x = 0 \qquad \cos^2 x = 1$

$\cos x = \pm 1$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Find <u>all</u> the solutions.

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$

$\sec 3x = \frac{2\sqrt{3}}{3}$

$\sec(3x) = \frac{2}{\sqrt{3}}$

$3x = \frac{\pi}{6} + 2\pi k, \quad 3x = \frac{11\pi}{6} + 2\pi k$

$x = \frac{\pi}{18} + \frac{2\pi k}{3}$ & $x = \frac{11\pi}{18} + \frac{2\pi k}{3}$

68. $\cos\left(\underbrace{2x - \frac{\pi}{4}}_{\theta}\right) = -\frac{\sqrt{2}}{2}$

$\cos\theta = -\frac{1}{\sqrt{2}}$

$\theta = \frac{3\pi}{4} + 2\pi k$ & $\frac{\cancel{3}5\pi}{4} + 2\pi k$

$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$

$x = \frac{\pi}{2} + \pi k$

$2x - \frac{\pi}{4} = \frac{\cancel{3}5\pi}{4} + 2\pi k$

$2x = \frac{6\pi}{4} + 2\pi k$

$x = \frac{3\pi}{4} + \pi k$

6.6 handout

1-21 odd $[0, 2\pi)$

&

61-69 odd (all x's)