Verify the identity.

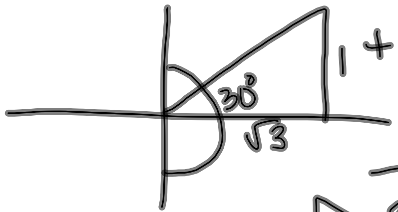
1. $\frac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

$$LHS = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} = \cot^2 x - 1 = RHS$$

Evaluate the inverse trigonometric function.

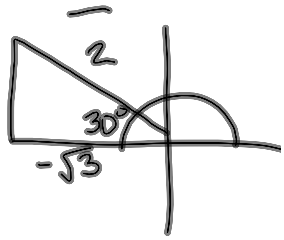
2. $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) =$ $\frac{\pi}{6}$

$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$




3. $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right) =$ $\frac{5\pi}{6}$

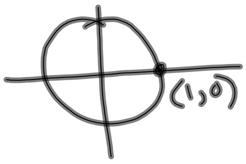
$0 \leq \theta \leq \pi$




Solve for $x \in [0, 2\pi)$.

4. $\cos x = 1$

$\boxed{x = 0}$




5. $\sin 2x = 0$ $\longrightarrow$ $0 \leq x < 2\pi$, $0 \leq 2x < 4\pi$

$2\sin x \cos x = 0$

$2\sin x = 0$ $\quad \cos x = 0$

$\sin x = 0$ $\quad x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = 0, \pi$

$2x = 0, \pi, 2\pi, 3\pi$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

Bonus: Solve for $x \in [0, 2\pi)$. $\cot^2 4x = \frac{1}{3}$

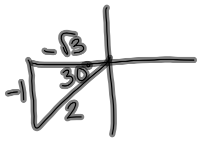
$\cot^2 4x = \frac{1}{3}$

$\cot 4x = \pm\frac{1}{\sqrt{3}}$

$4x = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}, \frac{10\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}, \frac{16\pi}{3}, \frac{17\pi}{3}, \frac{19\pi}{3}, \frac{20\pi}{3}, \frac{22\pi}{3}, \frac{23\pi}{3}$

$x = \frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{12}, \frac{7\pi}{12}, \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{11\pi}{12}, \frac{13\pi}{12}, \frac{7\pi}{6}, \frac{4\pi}{3}, \frac{17\pi}{12}, \frac{19\pi}{12}, \frac{5\pi}{3}, \frac{11\pi}{6}, \frac{23\pi}{12}$

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.




$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\tan\frac{7\pi}{12} = \tan\left(\frac{\frac{7\pi}{6}}{2}\right) = \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}} =$$

$$\frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}} = \left(1+\frac{\sqrt{3}}{2}\right)\cdot\frac{-2}{1} = \boxed{-2-\sqrt{3}}$$

$\cos a \cos b + \sin a \sin b = \cos(a-b)$

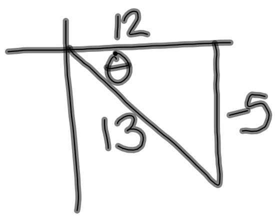
2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$$= \cos(212° - 122°)$$

$$= \cos 90°$$

$$= \boxed{0}$$

3. Find $\sin 2\theta, \cos 2\theta,$ *and* $\tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and $\theta$ is in Quadrant IV.



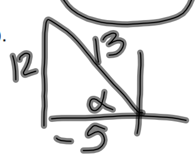

$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \frac{-120}{169}$$

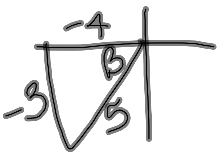
$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144-25}{169} = \boxed{\frac{119}{169}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120/169}{119/169} = \frac{-120}{\cancel{169}}\cdot\frac{\cancel{169}}{119} = \boxed{\frac{-120}{119}}$$

What Quadrant is $2\theta$ in? IV

4. Given $\sin\alpha = \frac{12}{13}$, $\alpha$ is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and $\beta$ is in Quadrant III, find $\sin(\alpha+\beta)$.

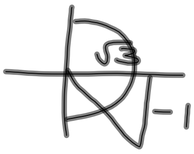



$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{12}{13}\right)\left(\frac{-4}{5}\right) + \left(\frac{-5}{13}\right)\left(\frac{-3}{5}\right)$$
$$= \frac{-48}{65} + \frac{15}{65} = \boxed{\frac{-33}{65}}$$

5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

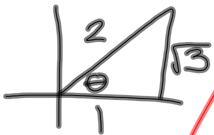



$= \boxed{-\frac{\pi}{6}}$

$-\frac{\pi}{2}, \frac{\pi}{2}$ sin, csc, tan

$0, \pi$ cos, sec, cot

6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right)$




$\boxed{\frac{1}{2}}$

$\cot^{-1}(\cos\pi)$

$\cot^{-1}(-1)$

$\boxed{\frac{3\pi}{4}}$

7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$\sin^2 x = \frac{1}{4}$

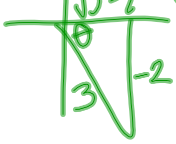
$\sin x = \pm\frac{1}{2}$

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$\sec\left(\underbrace{\sin^{-1}\left(-\frac{2}{3}\right)}_{\theta}\right)$

$= \boxed{\frac{3}{\sqrt{5}}}$

$\sqrt{3^2 - 2^2} = \sqrt{5}$




8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$2\sin^3 x - \sin x = 0$

$\sin x(2\sin^2 x - 1) = 0$

$\sin x = 0$ $\quad$ $2\sin^2 x - 1 = 0$

$x = 0, \pi$ $\quad$ $\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$

$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. Prove the identity.

$$\frac{1 + \cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$LHS = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \csc^2 x + \cot^2 x$

$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

$= \csc^2 x + (\csc^2 x - 1)$

$= 2\csc^2 x - 1 = RHS$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$
$$= \frac{1}{\sin x} - \frac{\cos^2 x}{\sin x}$$
$$= \frac{1-\cos^2 x}{\sin x}$$
$$= \frac{\sin^2 x}{\sin x} = \frac{\sin x \cancel{\sin x}}{\cancel{\sin x}} = RHS$$

$\sin^2 x + \cos^2 x = 1$
$\sin^2 x = 1 - \cos^2 x$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \le x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$

$\sin(2x+x) + \sin x - \sin 2x = 0$
$\sin 2x \cos x + \cos 2x \sin x + \sin x - \sin 2x = 0$
$(2\sin x \cos x)\cos x + (\cos^2 x - \sin^2 x)\sin x + \sin x - 2\sin x \cos x = 0$
$2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x + \sin x - 2\sin x \cos x = 0$
$3\sin x \cos^2 x - \sin^3 x + \sin x - 2\sin x \cos x = 0$
$3\sin x(1-\sin^2 x) - \sin^3 x + \sin x - 2\sin x \cos x = 0$
$3\sin x - 3\sin^3 x - \sin^3 x + \sin x - 2\sin x \cos x = 0$
$4\sin x - 4\sin^3 x - 2\sin x \cos x = 0$
$-2\sin x(2\sin^2 x - 2 + \cos x) = 0$

$-2\sin x = 0$
$x = 0, \pi$

$2(1-\cos^2 x) - 2 + \cos x$
$\cancel{2} - 2\cos^2 x \cancel{-2} + \cos x$
$\cos x(1 - \cos x) = 0$
$\cos x = 0 \quad \cos x = 1$
$x = \frac{\pi}{2}, \frac{3\pi}{2}$