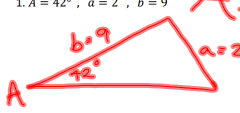**Part I.** Solve the given triangle. Circle your final answers, with side or angle the measurement corresponds to clearly indicated. If there is no solution, state this. If there is more than one solution, make sure to clearly indicate which answers go together.
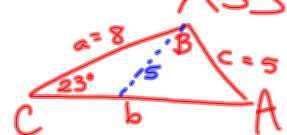
1. $A = 42°$ , $a = 2$ , $b = 9$    ASS

$$\frac{\sin B}{9} = \frac{\sin 42°}{2}$$

$$B = \sin^{-1}\left(\frac{9\sin 42°}{2}\right) = \emptyset$$

not a triangle

2. $a = 8$ , $c = 5$ , $C = 23°$    ASS

$$\frac{\sin A}{8} = \frac{\sin 23°}{5}$$

$$A = \sin^{-1}\left(\frac{8\sin 23°}{5}\right)$$

$$= \boxed{38.7°}$$

$$B = 180° - 23° - 38.7°$$
$$= \boxed{118.3°}$$

$$\frac{b}{\sin 118.3°} = \frac{5}{\sin 23°}$$

$$b = \frac{5\sin 118.3°}{\sin 23°} = \boxed{11.3}$$

case 2

$$A_2 = 180° - 38.7°$$
$$= \boxed{141.3°}$$

$$B_2 = 180° - 23° - 141.3°$$
$$= \boxed{15.7°}$$

$$\frac{b}{\sin 15.7°} = \frac{5}{\sin 23°}$$

$$b_2 = \frac{5\sin 15.7°}{\sin 23°}$$

$$= \boxed{3.5}$$
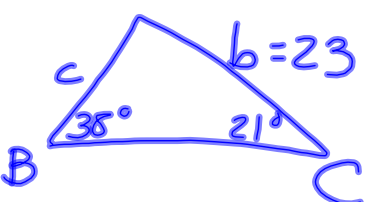
**Part II.** Find only the requested side or angle. Circle your final answer, with side or angle the measurement corresponds to clearly indicated. If there is no solution, state this. If there is more than one solution, circle both.

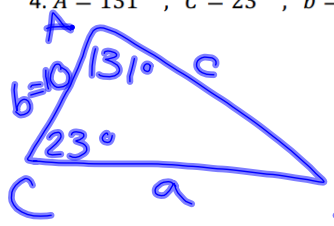3. $B = 38°$ , $C = 21°$ , $b = 23$   ; Find side c.    AAS

$$\frac{c}{\sin 21°} = \frac{23}{\sin 38°}$$

$$c = \frac{23\sin 21°}{\sin 38°} \approx \boxed{13.4}$$

4. $A = 131°$ , $C = 23°$ , $b = 10$  ; Find side a.    ASA

$$B = 180° - 131° - 23° = 26°$$

$$\frac{a}{\sin 131°} = \frac{10}{\sin 26°}$$

$$a = \frac{10\sin 131°}{\sin 26°} \approx \boxed{17.2}$$

1

5. $a = 12$ , $b = 14$ , $c = 20$ ; Find angle B.    SSS

$b = 14$

$b^2 = a^2 + c^2 - 2ac \cos B$

$a = 12$

$2ac \cos B = a^2 + c^2 - b^2$

B       $c = 20$
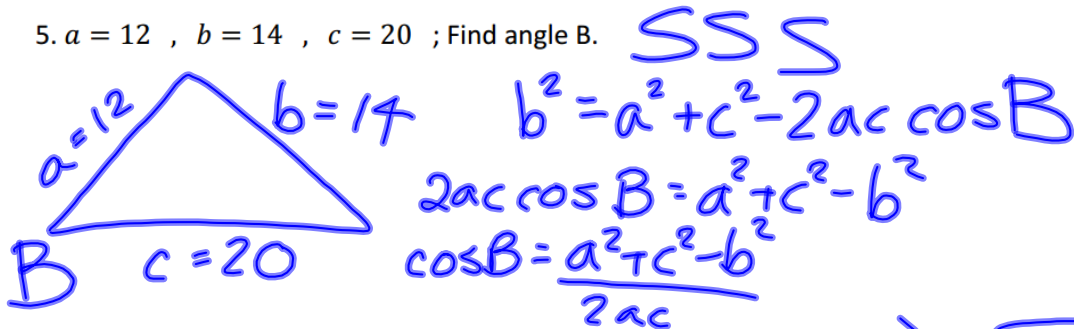
$\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

$B = \cos^{-1}\left(\dfrac{a^2 + c^2 - b^2}{2ac}\right) \approx \boxed{43.5°}$

6. $a = 15$ , $c = 7$ , $B = 62°$ ; Find side b.    SAS

$c = 7$       $b$

62°

B    $a = 15$
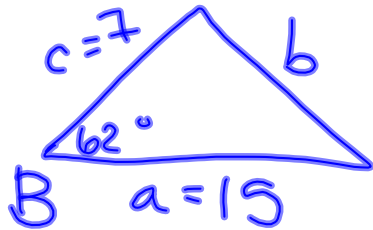
$b = \sqrt{a^2 + c^2 - 2ac \cos B}$

$= \sqrt{15^2 + 7^2 - 2(15)(7)\cos 62°}$

$\approx \boxed{13.2}$

**Part III**. Find the area of the given triangle. Your answer must include appropriate units.

7. $B = 42°$ , $a = 7.2 \, ft$ , $c = 3.4 \, ft$

$\text{Area} = \tfrac{1}{2}ab\sin C$

$K = \tfrac{1}{2}(7.2)(3.4)\sin 42°$

$= \tfrac{1}{2}bc\sin A$

$\approx \boxed{8.2 \; ft^2}$

$= \tfrac{1}{2}ac\sin B$

8. $A = 135.2°$ , $b = 46.12 \, ft$ , $c = 36.74 \, ft$

$K = \tfrac{1}{2}(46.12)(36.74)\sin 135.2°$

$\approx \boxed{597 \; ft^2}$

**Part IV**.  Find the distance or angle requested in the word problem by using the law of sines or cosines. Your answer must include appropriate units.

*bearing* ✓

9. A navigator on a ship sights a lighthouse at a bearing of N35°E.  After traveling 9 miles at a heading of 340°, the ship sights the lighthouse at a bearing of S80°E.  How far is the ship from the lighthouse at the second sighting?  *Hint: recall that heading is measured clockwise from north, and that N35°E is read as "35 degrees east of north."*

*heading*

$S_2$

60°

20°
80°

9 mi

70°
35°

$S_1$

340°

L

$180° - 55° - 60°$
$= 65°$

55°

$$\frac{x}{\sin 55°} = \frac{9}{\sin 65°}$$

$$x = \frac{9 \sin 55°}{\sin 65°}$$

$$\approx \boxed{8.1 \text{ mi}}$$

10. A regular pentagon is inscribed in a circle with a radius of 20 inches. Find the length of one side of the pentagon.

20

x

θ

r = 20 in

$$\theta = \frac{360°}{5} = 72°$$

SAS

$$x = \sqrt{20^2 + 20^2 - 2(20)(20)\cos 72°}$$

$$\approx \boxed{23.5 \text{ in}}$$

**Bonus:** The sides of a parallelogram are 28 centimeters and 40 centimeters. The shorter diagonal of the parallelogram is 48 centimeters. Find the length of the longer diagonal of the parallelogram. *Hint: The diagonals of a parallelogram bisect one another.*

$$40^2 = \left(\frac{x}{2}\right)^2 + 24^2 - 2\left(\frac{x}{2}\right)(24)\cos\theta$$

$$\cos\theta = \frac{\left(\frac{x}{2}\right)^2 + 24^2 - 40^2}{2\left(\frac{x}{2}\right)(24)}$$

$$28^2 = \left(\frac{x}{2}\right)^2 + 24^2 - 2\left(\frac{x}{2}\right)(24)\cos(180^\circ-\theta)$$

$$\cos(180^\circ-\theta) = \frac{\left(\frac{x}{2}\right)^2 + 24^2 - 28^2}{2\left(\frac{x}{2}\right)(24)}$$

$$\cos(180^\circ - \theta)$$
$$= \cos 180^\circ \cos\theta + \sin 180^\circ \sin\theta$$
$$= -\cos\theta$$

$$\frac{\left(\frac{x}{2}\right)^2 + 24^2 - 40^2}{2\left(\frac{x}{2}\right)(24)} = -\frac{\left(\frac{x}{2}\right)^2 + 24^2 - 28^2}{2\left(\frac{x}{2}\right)(24)}$$

$$\frac{\frac{x^2}{4} + 24^2 - 40^2}{24x} = -\frac{\left(\frac{x^2}{4} + 24^2 - 28^2\right)}{24x}$$

$$\frac{x^2}{4} + \frac{x^2}{4} = -24^2 + 28^2 - 24^2 + 40^2$$

$$x^2 = 2(-24^2 + 28^2 - 24^2 + 40^2)$$

$$x = \sqrt{2(-24^2 + 28^2 - 24^2 + 40^2)}$$

$$= \boxed{49.6 \text{ cm}}$$