Review:

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

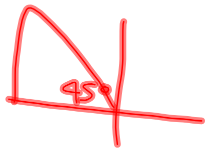
$\tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

Use the half-angle identity to evaluate $\tan\frac{3\pi}{8}$ exactly.

$$\tan\frac{\left(\frac{3\pi}{4}\right)}{2} = \frac{1-\cos\frac{3\pi}{4}}{\sin\frac{3\pi}{4}}$$

$$= \frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{\frac{1}{\sqrt{2}}}$$

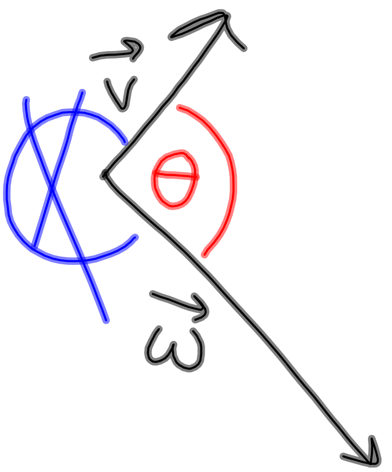


$\sin 45° = \cos 45°$
$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$$\left(1+\frac{\sqrt{2}}{2}\right)\cdot\frac{\sqrt{2}}{1} = \boxed{\sqrt{2}+1}$$

The smallest nonnegative angle between two vectors is given by

$\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}$, or equivalently, $\boxed{\theta = \cos^{-1}\frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}}$.



7.6 $\boxed{\cos\theta = \frac{\vec{v}\cdot\vec{w}}{|\vec{v}||\vec{w}|}}$

64. $\vec{a} = \langle -3,-3 \rangle$; $\vec{b} = \langle -5, 2 \rangle$

$\vec{a}\cdot\vec{b} = (-3)(-5) + (-3)(2)$
$= 15 - 6 = 9$

$|\vec{a}| = \sqrt{(-3)^2+(-3)^2} = 3\sqrt{2}$

$|\vec{b}| = \sqrt{(-5)^2+2^2} = \sqrt{29}$

$\cos\theta = \frac{9}{3\sqrt{2}\cdot\sqrt{29}}$

$\theta = \cos^{-1}\left(\frac{9}{3\sqrt{2}\sqrt{29}}\right) \approx \boxed{66.8^\circ}$

68. $\vec{u} = 3\vec{i} + 2\vec{j}$; $\vec{v} = -\vec{i} + 4\vec{j}$

$\vec{u}\cdot\vec{v} = 3(-1) + 2(4) = 5$

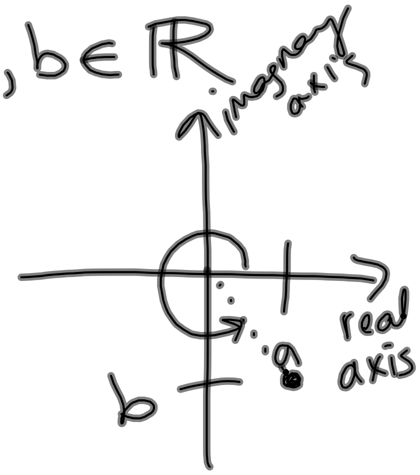
$|\vec{u}| = \sqrt{3^2+2^2} = \sqrt{13}$

$|\vec{v}| = \sqrt{(-1)^2+4^2} = \sqrt{17}$

$\theta = \cos^{-1}\left(\frac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}\right) = \cos^{-1}\left(\frac{5}{\sqrt{13}\sqrt{17}}\right) \approx \boxed{70.3^\circ}$

7.3 Trigonometric Form of Complex Numbers

z = a + bi , where i = $\sqrt{-1}$; $a, b \in \mathbb{R}$



a is the "real component"
b is the "imaginary component

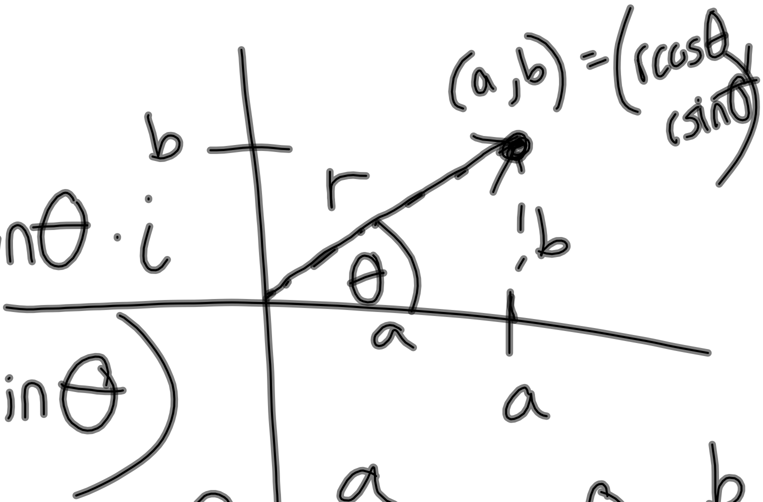
modulus $|z| = \sqrt{a^2+b^2}$

argument θ (counter-clockwise from 0)

~~r cis~~

~~mult/div~~

$z = a + bi$



$z = r\cos\theta + r\sin\theta \cdot i$

$= r(\cos\theta + i\sin\theta)$

$z = r\,\mathrm{cis}\,\theta$

$\cos\theta = \frac{a}{r}$ $\sin\theta = \frac{b}{r}$

$a = r\cos\theta$ $b = r\sin\theta$

$z_1 = r_1 \operatorname{cis}\theta_1 \; ; \; z_2 = r_2 \operatorname{cis}\theta_2$

$z_1 z_2 = (r_1\cos\theta_1 + ir_1\sin\theta_1)(r_2\cos\theta_2 + ir_2\sin\theta_2)$

$= r_1r_2\cos\theta_1\cos\theta_2 + ir_1r_2\cos\theta_1\sin\theta_2 +$
$\quad ir_1r_2\sin\theta_1\cos\theta_2 \cancel{+} \underbrace{\cancel{i^2}}_{i^2=-1} r_1r_2\sin\theta_1\sin\theta_2$

$= r_1r_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2) +$
$\quad ir_1r_2(\cos\theta_1\sin\theta_2 + \sin\theta_1\cos\theta_2)$

$= r_1r_2\cos(\theta_1+\theta_2) + ir_1r_2\sin(\theta_1+\theta_2)$

$= r_1r_2 \operatorname{cis}(\theta_1+\theta_2)$

To multiply two complex #'s in trigonometric form multiply the moduli & add the arguments.

$z_1 = -2\operatorname{cis}20^\circ \; ; \; z_2 = 5\operatorname{cis}40^\circ$

$z_1z_2 = \boxed{-10\operatorname{cis}60^\circ}$

$z_1 = r_1\operatorname{cis}\theta_1 \; ; \; z_2 = r_2\operatorname{cis}\theta_2$

$$\frac{z_1}{z_2} = \frac{r_1}{r_2}\operatorname{cis}(\theta_1 - \theta_2)$$

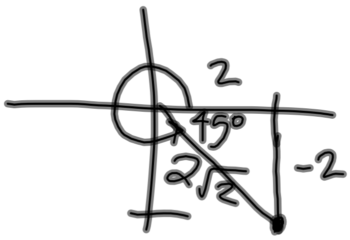
$z_1 = 12\operatorname{cis}120^\circ \; ; \; z_2 = -4\operatorname{cis}10^\circ$

$$\frac{z_1}{z_2} = \boxed{-3\operatorname{cis}110^\circ}$$

converting between
standard form & trigonometric form
$a+bi$ $r \text{ cis } \theta$

$$z = 5 \text{ cis } 30^\circ = 5\cos 30^\circ + 5i\sin 30^\circ$$
$$= 5\left(\sqrt{3}/2\right) + 5i\left(1/2\right)$$
$$= \boxed{\frac{5\sqrt{3}}{2} + \frac{5}{2}i}$$

$$z = 2 - 2i$$
$$= 2\sqrt{2} \text{ cis } 315^\circ$$



$r = 2\sqrt{2}$

$\theta = 315^\circ$

7.6

#63-67

7.3

#13-43odd