

Convert 120° to radians.

$$120^\circ \cdot \frac{\pi}{180^\circ} = \boxed{\frac{2\pi}{3}}$$

Convert $\frac{7\pi}{4}$ to degrees.

$$\frac{7\pi}{4} \cdot \frac{180^\circ}{\pi} = \boxed{315^\circ}$$

Two angles in radians are:

complementary if they sum to $\frac{\pi}{2} \equiv 90^\circ$

supplementary if they sum to $\pi \equiv 180^\circ$

coterminal if they differ by integer multiples of $2\pi \equiv 360^\circ$

Find the complement and supplement of $\frac{5\pi}{12}$.

$$C: \frac{\pi}{2} - \frac{5\pi}{12} = \frac{6\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{\pi}{12}}$$

$$S: \pi - \frac{5\pi}{12} = \frac{12\pi}{12} - \frac{5\pi}{12} = \boxed{\frac{7\pi}{12}}$$

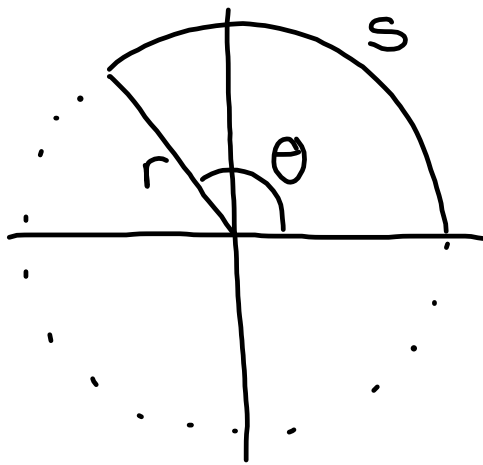
$$C: \frac{\pi}{2} - 0.5$$

$$S: \pi - 0.5$$

Find one positive and one negative angle coterminal with $-\frac{3\pi}{4}$.

$$\frac{-3\pi}{4} + 2\pi = \frac{-3\pi}{4} + \frac{8\pi}{4} = \boxed{\frac{5\pi}{4}}; \quad \frac{-3\pi}{4} - \frac{8\pi}{4} = \boxed{\frac{-11\pi}{4}}$$

Arc Length & Angular Speed



Arc Length

r = radius or distance from the center of rotation
(in, cm, km, etc.)

s = arc length or distance traveled along the circumference of a circle
(in, cm, km, etc.)

θ = angle or amount of rotation
(deg, rad, revolutions, etc.)

$$s = r\theta$$

1. $r = 5\text{in}$; $\theta = 45^\circ$; $s = ?\text{in}$

$$s = r\theta$$

$$= \frac{5\text{in}}{1} \cdot \frac{45^\circ}{1} \cdot \frac{\pi}{\cancel{180^\circ}} = \frac{5\pi}{4}\text{in}$$

2. $s = 16\text{yards}$; $\theta = 5$; $r = ?\text{yards}$

$$\frac{s}{\theta} = \frac{r\theta}{\theta}$$

$$r = \frac{s}{\theta} = \frac{16\text{yd}}{5}$$

3. Find the measure of a rotation in radians when a point 2 meters from the center of rotation travels 4 meters.

$$\theta = ? \text{ rad} ; r = 2\text{m} ; s = 4\text{m}$$

$$\frac{s}{r} = \frac{r\theta}{r}$$

$$\theta = \frac{s}{r} = \frac{4\text{m}}{2\text{m}} = \boxed{2}$$

Linear Speed

$$v = \frac{s}{t}$$

Angular Speed

$$\omega = \frac{\theta}{t}$$

$\omega = \text{"omega"}$

Arc Length

$$s = r\theta$$

Relating Linear & Angular Speed

$$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$$

$$\boxed{v = r\omega}$$

r = radius or distance from the center of rotation
(in, cm, km, etc.)

s = arc length or linear distance along the circumference of a circle
(in, cm, km, etc.)

θ = angle or amount of rotation
(deg, rad, revolutions, etc.)

t = time
(sec, min, hours, years, etc.)

$v = \frac{\text{linear distance}}{\text{time}} = \text{linear speed}$
($\frac{\text{km}}{\text{s}}, \frac{\text{mi}}{\text{h}}, \text{etc.}$)

$\omega = \frac{\text{amount of rotation}}{\text{time}} = \text{angular speed}$
($\frac{\text{rev}}{\text{min}}, \frac{\text{deg}}{\text{s}}, \text{etc.}$)

Handout Problems:

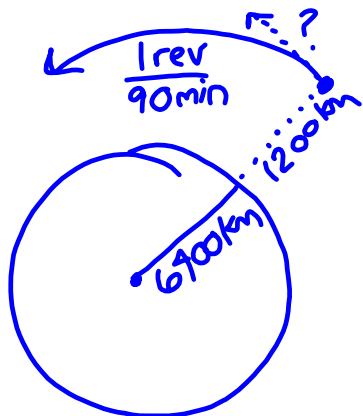
1. A wheel with a 15 inch diameter rotates at a rate of 6 radians per second. What is the linear speed of a point on its rim in feet per minute?

$$r = \frac{15 \text{ in}}{2} ; \omega = \frac{6 \text{ rad}}{s} ; v = ? \text{ ft/min}$$

$$v = r\omega = \frac{15 \text{ in}}{2} \cdot \frac{6 \text{ rad}}{s} \cdot \frac{1 \text{ ft}}{12 \text{ in}} \cdot \frac{60 \text{ s}}{1 \text{ min}} =$$

$$= 225 \text{ ft/min}$$

2. An earth satellite in circular orbit 1200 km high makes one complete revolution every 90 minutes. What is its linear speed in km/min, given that the earth's radius is 6400 km?



$$r = 7600 \text{ km}$$

$$\omega = \frac{1 \text{ rev}}{90 \text{ min}}$$

$$v = ? \frac{\text{km}}{\text{min}}$$

$$v = r\omega = \frac{7600 \text{ km}}{1} \cdot \frac{1 \text{ rev}}{90 \text{ min}} \cdot \frac{2\pi}{1 \text{ rev}} = \frac{1520\pi \text{ km}}{9 \text{ min}}$$

3. Through how many radians does the minute hand of a clock rotate from 12:45pm to 1:25pm?

$$\theta = ? \text{ rad} ; t = 40 \text{ min} ; \omega = \frac{1 \text{ rot}}{1 \text{ h}}$$

$$t \cdot \omega = \frac{\theta}{t} \cdot t$$

$$\theta = \omega \cdot t = \frac{1 \text{ rot}}{1 \text{ h}} \cdot \frac{40 \text{ min}}{1} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{2\pi}{1 \text{ rot}}$$

$$= \boxed{\frac{4\pi}{3}}$$

4. A car travels at 60 miles per hour. Its wheels have a 24 inch diameter. What is the angular speed of a point on the rim of a wheel in revolutions per minute?

$$v = \frac{60 \text{ mi}}{\text{h}} ; r = 12 \text{ in} ; \omega = ? \frac{\text{rev}}{\text{min}}$$

$$\frac{v}{r} = \frac{r\omega}{r}$$

$$\omega = \frac{v}{r} = \frac{60 \text{ mi}}{\text{h}} \cdot \frac{1}{12 \text{ in}}$$

$$\omega = \frac{v}{r} = \frac{v}{1} \cdot \frac{1}{r}$$

$$\omega = \frac{60 \text{ mi}}{\text{h}} \cdot \frac{1}{12 \text{ in}} \cdot \frac{1 \text{ h}}{60 \text{ min}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{1 \text{ rev}}{2\pi}$$

$$= \boxed{\frac{2640 \text{ rev}}{\pi \text{ min}}}$$

Homework due this Friday:

Already assigned:

- 5.1 #1, 2, 7-18 all, 31-54 all

New:

- **4 problems on handout**
- **5.1 #55-74 all**

Due next Wednesday, 11/13:

"Do you know enough Algebra..." take-home quiz