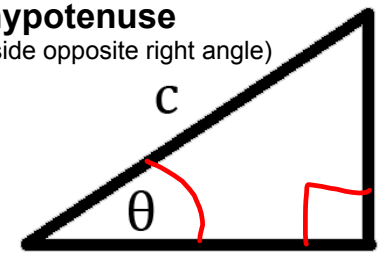
5.2 Trigonometric Functions of Acute Angles

An acute angle is an angle between 0° & 90° .

A right triangle is a triangle with a 90° angle.



hypotenuse
(side opposite right angle)

opposite side
(side opposite angle of interest)

adjacent side
(side adjacent to angle of interest)

θ theta

α alpha

β beta

γ gamma

The six basic trigonometric functions are ratios of sides of a right triangle.

sine $\sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse} = \frac{opp}{hyp}$

cosine $\cos\theta = \frac{length\ of\ side\ adjacent\ to\ \theta}{length\ of\ hypotenuse} = \frac{adj}{hyp}$

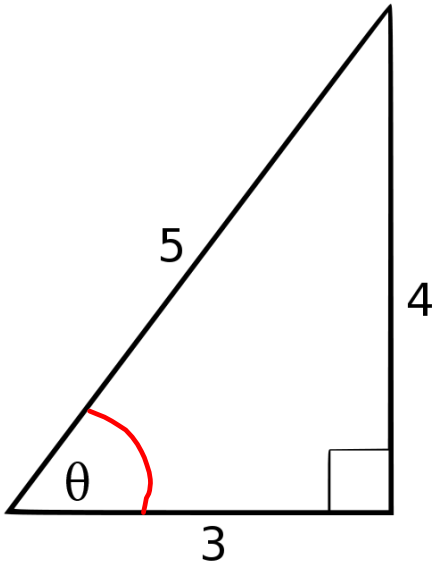
tangent $\tan\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ to\ \theta} = \frac{opp}{adj}$

SohCahToa

secant $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

cosecant $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

cotangent $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta}$



$\sin\theta = 4/5$

$\cos\theta = 3/5$

$\tan\theta = 4/3$

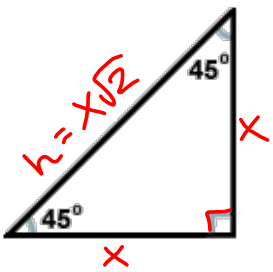
$\sec\theta = 5/3$

$\csc\theta = 5/4$

$\cot\theta = 3/4$

Two special right triangles:

Isosceles Right Triangle aka 45-45-90

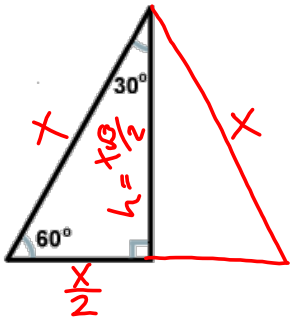


$x^2 + x^2 = h^2$

$2x^2 = h^2$

$\sqrt{2}\cdot x = h$

Half of an equilateral triangle aka 30-60-90



Pythagorean Theorem:

$a^2 + b^2 = c^2$

a, b = legs

c = hypotenuse

$1 < \sqrt{3} < 2$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

$1 < 3 < 4$

$\left(\frac{x}{2}\right)^2 + h^2 = x^2$

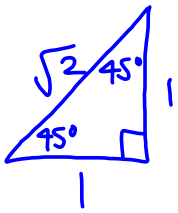
$\frac{x^2}{4} + h^2 = x^2$

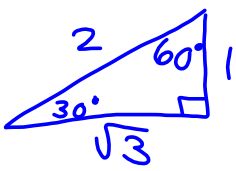
$h^2 = x^2 - \frac{x^2}{4}$

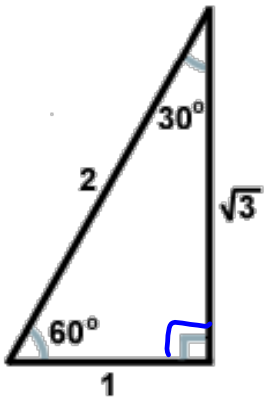
$h^2 = \frac{4x^2}{4} - \frac{x^2}{4}$

$h^2 = \frac{3x^2}{4}$

$h = \frac{\sqrt{3}}{2}\cdot x$







$\sin 30° = \frac{1}{2}$ $\quad$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$ $\quad$ $\tan 60° = \sqrt{3}$

$\sec 30° = \frac{2}{\sqrt{3}}$ $\quad$ $\sec 60° = 2$

$\csc 30° = 2$ $\quad$ $\csc 60° = \frac{2}{\sqrt{3}}$

$\cot 30° = \sqrt{3}$ $\quad$ $\cot 60° = \frac{1}{\sqrt{3}}$

30° & 60° are complementary

Cofunctions

The function of an angle is equal to the cofunction of its complement.

The complement of an angle θ is $90° - \theta$ or $\frac{\pi}{2} - \theta$

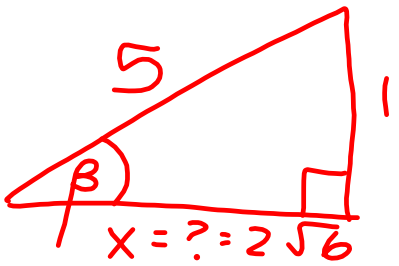
Examples:

$\cos(20°) = \sin(90° - 20°) = \sin 70°$

$\csc(89°) = \sec 1°$

$\tan(7°) = \cot 83°$

Given that $\csc\beta = 5$, find the other trigonometric function values of β.



$x^2 + 1^2 = 5^2$

$x^2 = 25 - 1$

$x^2 = 24$

$x = \sqrt{24}$

$x = 2\sqrt{6}$

$\sin\beta = 1/5$

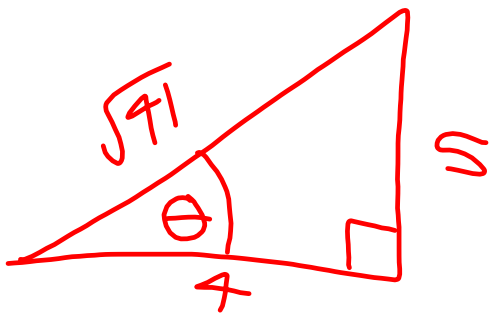
$\cos\beta = 2\sqrt{6}/5$

$\tan\beta = 1/2\sqrt{6}$

$\sec\beta = 5/2\sqrt{6}$

$\cot\beta = 2\sqrt{6}$

Given that $\cot\theta = \frac{4}{5}$, find the other trigonometric function values of θ.



$\sin\theta = 5/\sqrt{41}$

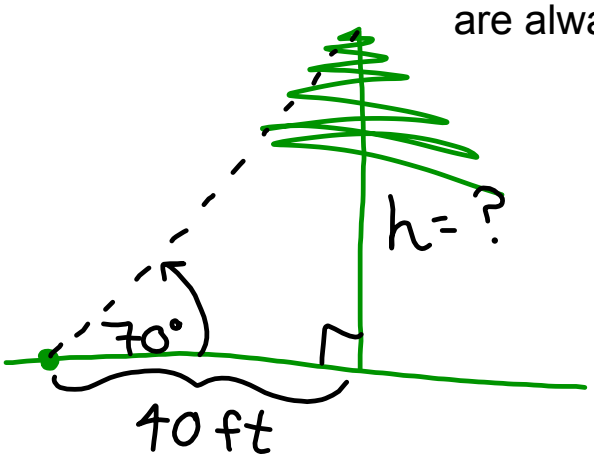
$\sec\theta = \sqrt{41}/4$

$\tan\theta = 5/4$

5.2 Applications of Right Triangles

A botanist stands 40 ft. from the base of a tree and estimates the angle of elevation to the tree's peak to be 70 degrees. How tall is the tree?

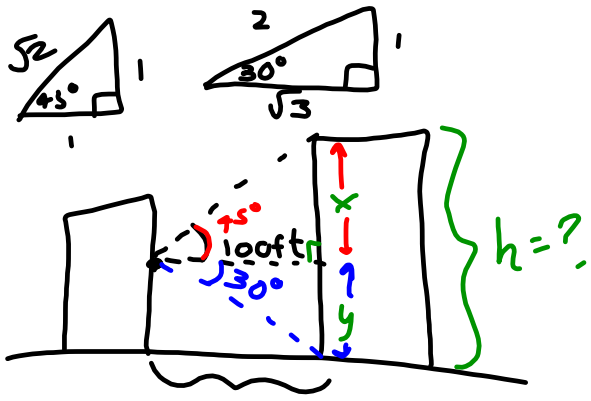
** Angles of elevation and depression are always measured from the horizontal



$$\tan 70^\circ = \frac{h}{40\text{ ft}}$$

$$h = 40\tan 70^\circ \text{ ft}$$

A window washer on the side of one building, 100 feet from another building, measures the angle of elevation of the top of the other building to be 45 degrees, and the angle of depression to the bottom of the other building to be 30 degrees. How tall is the other building?



$$\tan 45^\circ = \frac{x}{100}$$

$$x = 100\tan 45^\circ$$

$$\tan 30^\circ = \frac{y}{100}$$

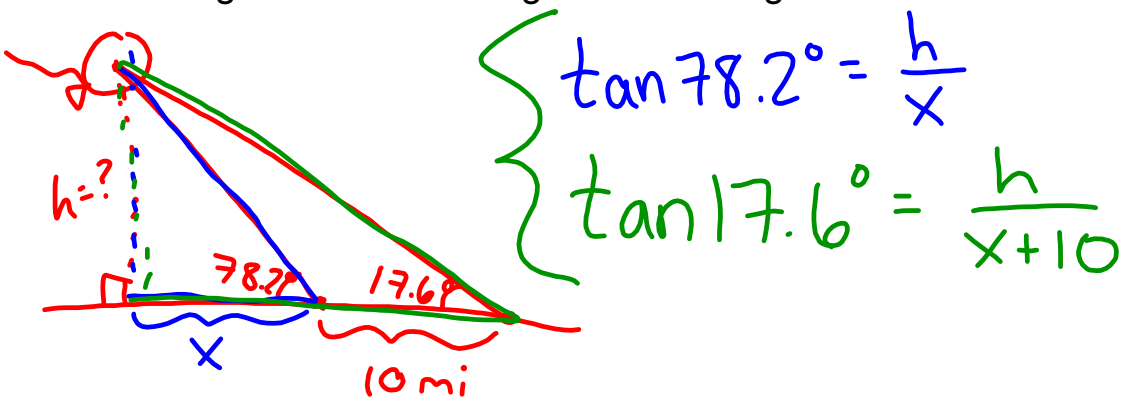
$$y = 100\tan 30^\circ$$

$$h = 100\tan 45^\circ + 100\tan 30^\circ \text{ ft}$$

$$= 100\cdot 1 + 100\cdot \tfrac{1}{\sqrt{3}}$$

$$h = 100 + 100/\sqrt{3} \text{ ft}$$

A weather balloon is directly west of two observing stations that are 10 miles apart. The angles of elevation of the balloon from the two stations are 17.6 degrees and 78.2 degrees. How high is the balloon?



Homework due this Friday:
Already assigned:

- 5.1 #1, 2, 7 18 all, 31 74 all
- 4 problems on handout

New:

- **5.2 #1-33odd**

Due next Wednesday, 11/13:

- "Do you know enough Algebra..." take home quiz

Due next Friday:

- **5.2 #35-75odd**