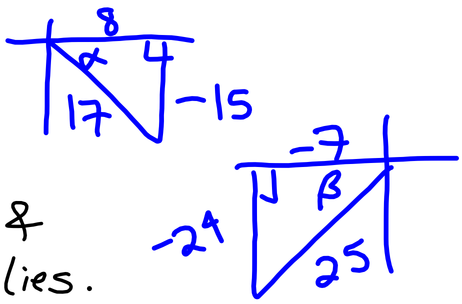
40. Given $\cos\alpha = \frac{8}{17}$, $\alpha \in QIV$

$\sin\beta = \frac{-24}{25}$, $\beta \in QIII$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.



$\sin(\alpha+\beta) =$

$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right) = \frac{105-192}{425} = \boxed{\frac{-87}{425}}$

*Pythagorean triples that are useful to know: 3,4,5 ; 5,12,13 ; 7,24,25 ; & 8,15,17

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

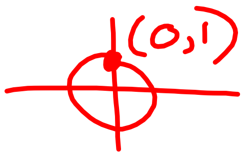
$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \frac{-56-360}{425} = \boxed{\frac{-416}{425}}$

$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{87/416}$ $\boxed{\alpha+\beta \in QIII}$

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90°-\theta$ or θ & $\frac{\pi}{2}-\theta$ are complementary angles



$\cos\left(\frac{\pi}{2}-x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$

$= 0\cdot\cos x + 1\cdot\sin x$

$= \boxed{\sin x}$

Double-Angle Identities

$\sin 2\theta = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 6\theta = \sin 2(3\theta) = \boxed{2\sin 3\theta\cos 3\theta}$

$\sin 8\theta = \sin 2(4\theta) = \boxed{2\sin 4\theta\cos 4\theta}$

$\sin 44\theta = \sin 2(22\theta) = \boxed{2\sin 22\theta\cos 22\theta}$

$\sin 3\theta = \sin 2\left(\frac{3\theta}{2}\right) = \boxed{2\sin\frac{3\theta}{2}\cos\frac{3\theta}{2}}$

$= \sin(2\theta + \theta)$

even multiple ⇒ double ∠'s

odd multiple ⇒ sum

$$\cos 2\theta = \cos(\theta+\theta)$$
$$= \cos\theta\cos\theta - \sin\theta\sin\theta$$
$$\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$$
$$= \cos^2\theta - (1-\cos^2\theta)$$
$$\boxed{= 2\cos^2\theta - 1}$$
$$= (1-\sin^2\theta) - \sin^2\theta$$
$$= \boxed{1 - 2\sin^2\theta}$$

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sin^2\theta = 1 - \cos^2\theta$$
$$\cos^2\theta = 1 - \sin^2\theta$$

$$\tan 2\theta = \tan(\theta+\theta)$$
$$= \frac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$$
$$\boxed{\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}}$$

Double-Angle Identities

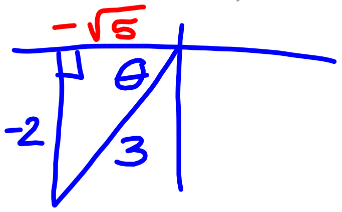
$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.



$$\sin 2\theta = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right) = \boxed{\frac{4\sqrt{5}}{9}}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$
$$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2 = \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}}$$

$$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{\frac{4\sqrt{5}}{9}}{\frac{1}{9}} = \frac{4\sqrt{5}}{\cancel{9}} \cdot \frac{\cancel{9}}{1} = \boxed{4\sqrt{5}}$$

$$2\theta \in \boxed{QI}$$

Half-Angle Identities

$\sin\frac{x}{2} = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$.

$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$

$\cos\frac{x}{2} = ?$

$\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = \frac{x}{2}$

$\cos 2\left(\frac{x}{2}\right) = 2\cos^2\frac{x}{2} - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

$\frac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\boxed{\pm\sqrt{\frac{1+\cos x}{2}} = \cos\frac{x}{2}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

$$\boxed{\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}}$$

Evaluate using the half-angle identity.

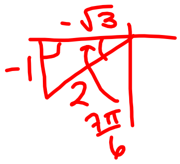
$$\tan\frac{7\pi}{12} = \tan\frac{\frac{7\pi}{6}}{2}$$

$$\frac{7\pi}{12} = \frac{1}{2}\cdot\boxed{\theta}$$

$$2\cdot\frac{7\pi}{12} = \theta$$

$$\frac{7\pi}{6} = \theta$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$



$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{-1}{2}}$$

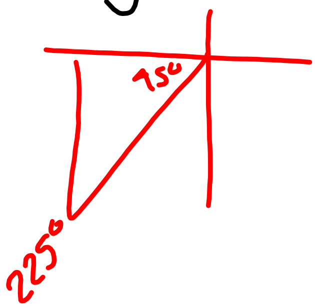
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$= \frac{1+\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{\frac{2}{2}+\frac{\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{\frac{2+\sqrt{3}}{2}}{\frac{-1}{2}} = \frac{2+\sqrt{3}}{\cancel{2}}\cdot\frac{-\cancel{2}}{1}$$

$$= \boxed{-2-\sqrt{3}}$$

6.3 Evaluate using the half-angle identity.

14. sin 112.5° $= \sin\frac{225^\circ}{2}$



$$= +\sqrt{\frac{1-\cos 225^\circ}{2}}$$

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$$

$$= \ldots$$

Homework #6:

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

& **memorize your identities!!!**