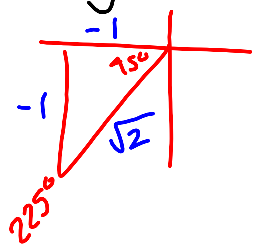
6.3 Evaluate using the half-angle identity.

14. $\sin 112.5^\circ = \sin\frac{225^\circ}{2}$



$= +\sqrt{\frac{1-\cos 225^\circ}{2}}$

$$\boxed{\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}}$$

$= \sqrt{\frac{1-\left(-\frac{\sqrt{2}}{2}\right)}{2}} = \sqrt{\frac{\frac{2}{2}1+\frac{\sqrt{2}}{2}}{2}}$

$= \sqrt{\frac{\frac{2+\sqrt{2}}{2}}{\frac{2}{1}}} = \sqrt{\frac{2+\sqrt{2}}{2}\cdot\frac{1}{2}} = \sqrt{\frac{2+\sqrt{2}}{4}} = \frac{\sqrt{2+\sqrt{2}}}{\sqrt{4}} =$

$= \boxed{\frac{\sqrt{2+\sqrt{2}}}{2}}$

$-\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$

6.3 Prove/Verify the identity.

50. $\cos 8x = \cos^2 4x - \sin^2 4x$

LHS $= \cos 8x = \cos 2(4x) = \cos^2 4x - \sin^2 4x =$

$= \text{RHS}\ \checkmark$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$
$= 2\cos^2\theta - 1$
$= 1 - 2\sin^2\theta$

$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$

$1 + \cot^2 x = \csc^2 x$

52. $\frac{\cos 2x}{\sin^2 x} = \cot^2 x - 1$

LHS $= \frac{\cos 2x}{\sin^2 x} = \frac{2\cos^2 x - 1}{\sin^2 x} = \frac{2\cos^2 x}{\sin^2 x} - \frac{1}{\sin^2 x} =$

$= 2\cot^2 x - \csc^2 x = 2\cot^2 x - (1+\cot^2 x)$

$= \cot^2 x - 1 = \text{RHS}\ \checkmark$

Better choice:

LHS $= \frac{\cos 2x}{\sin^2 x} = \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} =$

$= \cot^2 x - 1 = \text{RHS}\ \checkmark$

54. $\dfrac{1}{1-\cos 2x} = \dfrac{1}{2}\csc^2 x$

LHS $= \dfrac{1}{1-\cos 2x} = \dfrac{1}{1-(1-2\sin^2 x)} = \dfrac{1}{2\sin^2 x} = \dfrac{1}{2}\csc^2 x$

$=$ RHS ✓

56. $\dfrac{\cos^2 x - \sin^2 x}{2\sin x\cos x} = \cot 2x$

LHS $= \dfrac{\cos 2x}{\sin 2x} = \cot 2x =$ RHS ✓

60. $\sin 2x - \cot x = -\cot x\cos 2x$

LHS $= 2\sin x\cos x - \cot x$

$= 2\sin x\cos x - \dfrac{\cos x}{\sin x}$

$= 2\sin x\cos x \cdot \dfrac{\sin x}{\sin x} - \dfrac{\cos x}{\sin x}$

$= \dfrac{2\sin^2 x\cos x - \cos x}{\sin x}$

$= \dfrac{-\cos x(-2\sin^2 x + 1)}{\sin x} = -\dfrac{\cos x}{\sin x} \cdot \dfrac{1-2\sin^2 x}{1}$

$= -\cot x\cos 2x =$ RHS ✓

$\boxed{\dfrac{ab}{c} = \dfrac{a}{c} \cdot \dfrac{b}{1}}$

62. $\sin 4x = 4\sin x\cos^3 x - 4\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\underline{\sin 2x}\,\underline{\cos 2x} =$

$= 2(2\sin x\cos x)(\cos^2 x - \sin^2 x) =$

$= 4\sin x\cos x(\cos^2 x - \sin^2 x) =$

$= 4\sin x\cos^3 x - 4\sin^3 x\cos x =$

$= RHS$ ✓

64. $2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x = \cos^2 2x$

$RHS = (\cos 2x)^2 = (\cos 2x)(\cos 2x) =$

$= (\cos^2 x - \sin^2 x)(2\cos^2 x - 1) =$

$= 2\cos^4 x - \cos^2 x - 2\sin^2 x\cos^2 x + \sin^2 x =$

$= LHS$ ✓

66. $\sin 4x = 4\sin x\cos x - 8\cos x\sin^3 x$

$LHS = \sin 2(2x) = 2\sin 2x\cos 2x =$

$= 2(2\sin x\cos x)(1-2\sin^2 x) =$

$= 4\sin x\cos x(1-2\sin^2 x) =$

$= 4\sin x\cos x - 8\sin^3 x\cos x =$

$= RHS$ ✓

Homework #6 - Due Monday 01/13

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

& **memorize your identities!!!**