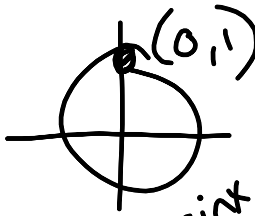
6.1

2. $\tan 2x = 2\tan x$



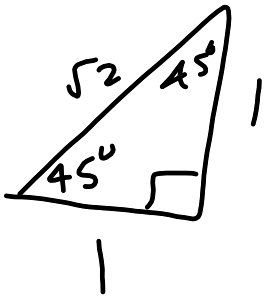
$\tan x = \frac{\sin x}{\cos x}$

$$\tan(2\cdot 45^\circ) \stackrel{?}{=} 2\tan(45^\circ)$$

$$\tan 90^\circ \stackrel{?}{=} 2\tan(45^\circ)$$

$$\frac{1}{0} \stackrel{?}{=} 2\cdot 1$$

$$\text{undef.} \stackrel{?}{=} 2$$



6.1

45. $$\frac{\frac{1}{\sin x}+1}{\frac{1}{\sin x}-1} = \tan^2 x + 2\tan x\sec x + \sec^2 x$$

$$LHS = \frac{\frac{1}{\sin x}+\frac{\sin x}{\sin x}}{\frac{1}{\sin x}-\frac{\sin x}{\sin x}} = \frac{\frac{1+\sin x}{\sin x}}{\frac{1-\sin x}{\sin x}} =$$

$$= \frac{1+\sin x}{\cancel{\sin x}} \cdot \frac{\cancel{\sin x}}{1-\sin x} = \frac{1+\sin x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x}$$

$\frac{6.3}{67.}$

$$\cos 3x - \cos x = 4\cos^3 x - 4\cos x$$

$LHS = \cos(2x + x) - \cos x =$

$= \cos 2x \cos x - \sin 2x \sin x - \cos x =$

$=$

$\frac{6.3}{89.}$ $2\tan\frac{x}{2} = \dfrac{\sin^2 x + 1 - \cos^2 x}{\sin x(1+\cos x)}$

$LHS = 2\left(\dfrac{1-\cos x}{\sin x}\right) \cdot \dfrac{1+\cos x}{1+\cos x} = \dfrac{2\cdot(1-\cos^2 x)}{\sin x(1+\cos x)}$

$= \dfrac{(1-\cos^2 x) + (1-\cos^2 x)}{\sin x(1+\cos x)} =$

6.1

27. $$\frac{\sin x - 2 + \frac{1}{\sin x}}{\sin x - \frac{1}{\sin x}} = \frac{\sin x - 1}{\sin x + 1}$$

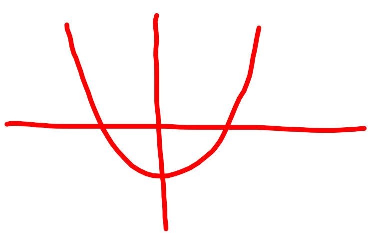
$$LHS = \frac{\left(\sin x - 2 + \frac{1}{\sin x}\right)}{\left(\sin x - \frac{1}{\sin x}\right)} \cdot \frac{\sin x}{\sin x} =$$

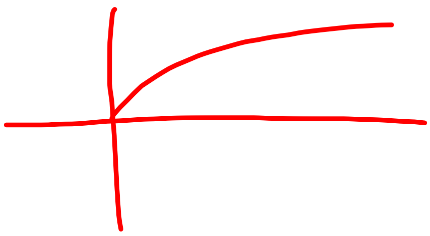
$$= \frac{\sin^2 x - 2\sin x + 1}{\sin^2 x - 1} =$$ factor & cancel

6.5 Inverse Trigonometric Functions

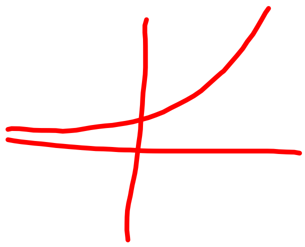
Recall from Algebra:

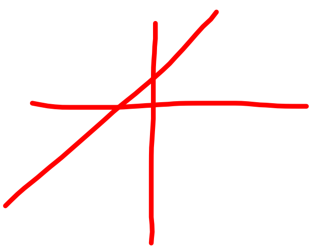
- f is a **function** if each input value (x) has a unique output $f(x)$.
  Examples: $f(x) = x^2 - 2$, $f(x) = \sqrt{x}$

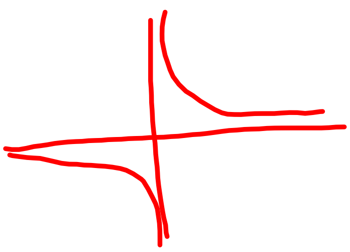


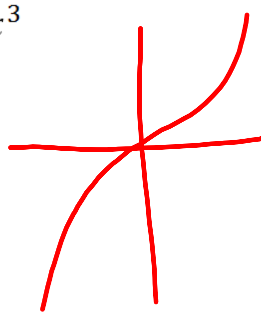


- f is **one-to-one** if, in addition, each y corresponds to only one x.
  Examples: $y = e^x$, $y = x + 3$, $y = \frac{1}{x}$, $y = x^3$









- If f is a one-to-one function, we can define its inverse $f^{-1}(x)$. Note that this notation is not exponentiation, i.e. $f^{-1}(x) \neq \frac{1}{f(x)}$

$x^{-n} = \frac{1}{x^n}$

- $f(x)$ and $g(x)$ are inverses if $(f \circ g)(x) = f(g(x)) = x = g(f(x)) = (g \circ f)(x)$, that is, **inverse functions "undo" each other**.

Example: $f(x) = x^3$, $g(x) = \sqrt[3]{x}$

$(f \circ g)(x) = (\sqrt[3]{x})^3 = x$

$(g \circ f)(x) = \sqrt[3]{x^3} = x$

What do we mean by an Inverse Trig function?

Recall that **for a basic Trigonometric function**, e.g. $f(x) = \sin x$,

- **The input (x) is an angle**
- **The output $f(x)$ is a ratio of sides**

So **for an inverse Trigonometric function,**

- **The input (x) is a ratio of sides**
- **The output $f(x)$ is an angle**

Construction of the inverse of $f(x) = \sin x$:

$f(x) = x^3 - 8$

$y = x^3 - 8$

$x = y^3 - 8$

$x + 8 = y^3$

$\sqrt[3]{x+8} = y$

$\boxed{f^{-1}(x) = \sqrt[3]{x+8}}$

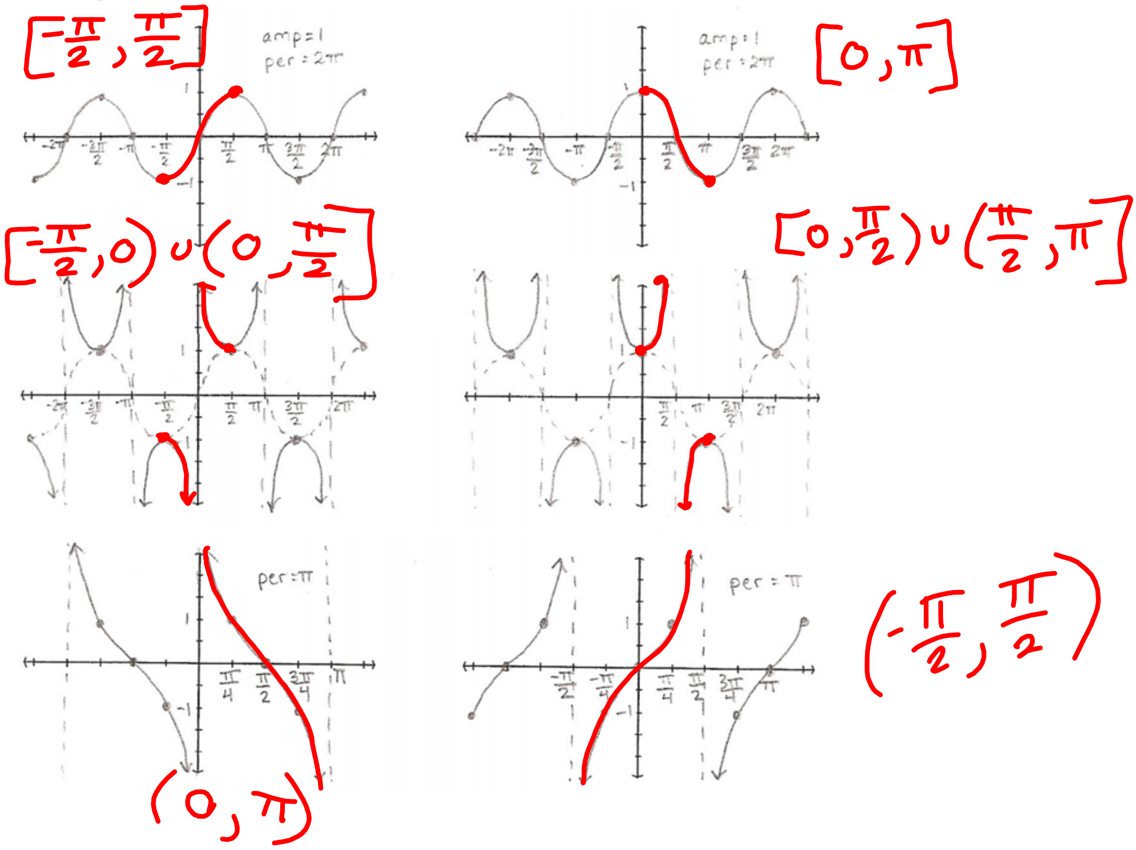
$y = \sin x$

$x = \sin y$

$y =$ the angle whose sine value is x

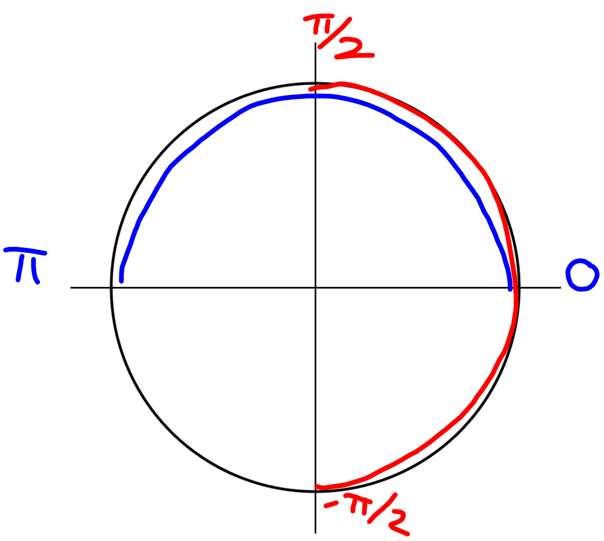
$y = \sin^{-1} x = \arcsin x$

But Trigonometric functions aren't one-to-one – how is the inverse defined? We must restrict the domain!



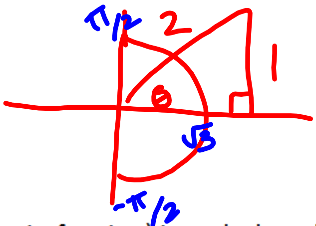
Summary of Restricted Domains:

Interval	Functions	Quadrants
$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$	$\sin x, \csc x, \tan x$	IV & I
$(0, \pi)$	$\cos x, \sec x, \cot x$	I & II



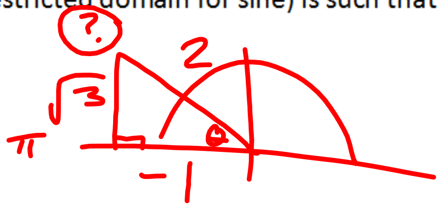
Evaluate the inverse trigonometric expression.

$\sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$



In words: What angle θ, between $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ (the restricted domain for sine) is such that $\sin\theta = \frac{1}{2}$?

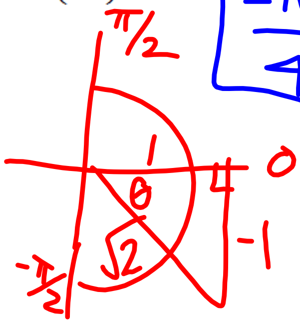
$\cos^{-1}\left(-\frac{1}{2}\right) = \boxed{\frac{2\pi}{3}}$



In words: What angle θ, between 0 and π (the restricted domain for cosine) is such that $\cos\theta = -\frac{1}{2}$?

What angle θ, between $-\pi/2$ & $\pi/2$, is such that $\tan\theta = -1$?

$\tan^{-1}(-1) = \boxed{-\frac{\pi}{4}}$

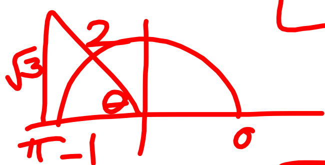


$\frac{7\pi}{4}$ is $\frac{\pi}{4}$ in QIV

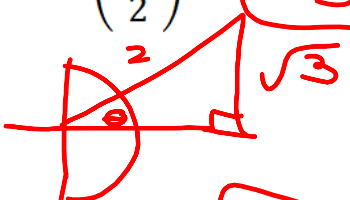
$-\frac{\pi}{2} \le \cancel{\frac{7\pi}{4}} \le \frac{\pi}{2}$

Evaluate.

$\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = \boxed{\frac{2\pi}{3}}$

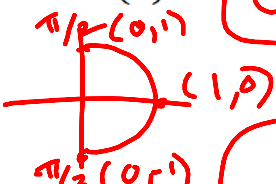


$\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \boxed{\pi/3}$

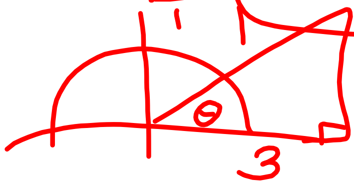


$\csc^{-1}(-2) = \boxed{-\frac{\pi}{6}}$

$\tan^{-1}(0) = \boxed{0}$



$\cos^{-1}(3) = \boxed{\text{undefined}}$



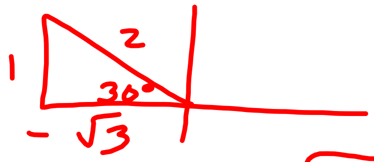
What happens when we compose a Trigonometric function with its inverse?

According to the definition,

$f(x)$ and $g(x)$ are inverses if $f\big(g(x)\big) = x$ and $g\big(f(x)\big) = x$
(for all x-values in the respective domains of g and f)

We would then expect

$\sin(\sin^{-1} x) = x$ and $\sin^{-1}(\sin x) = x$



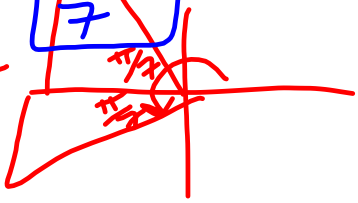
$\sin\left(\sin^{-1}\frac{1}{2}\right) = \sin 30° = \boxed{\frac{1}{2}}$

$\sin^{-1}\left(\sin\left(\frac{5\pi}{6}\right)\right) = \sin^{-1}\left(\frac{1}{2}\right) = \boxed{\frac{\pi}{6}}$

$\sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right) = \boxed{\frac{-\pi}{6}}$

$\cos^{-1}\left(\cos\left(\frac{8\pi}{7}\right)\right) = \boxed{\frac{6\pi}{7}}$

$\frac{7\pi}{7} = \pi$



$\sin(\sin^{-1} 3) = \boxed{\text{undefined}}$

Homework #6 - Due Wednesday

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

New Homework (not due Wednesday)
- **6.5 #1-24 all; 25-55 odd**

& **memorize your identities!!!**