6.6

Solving Trigonometric Equations

evaluate the expression

$\sin^{-1}\left(\frac{1}{2}\right)$ versus $\sin x = \frac{1}{2}$

solve the equation

one solution;

$\frac{\pi}{6}$

infinitely many solutions;

$\frac{\pi}{6} + 2\pi k, \ k \in \mathbb{Z}$

$\frac{5\pi}{6} + 2\pi k, \ k \in \mathbb{Z}$

$\in$ = "is an element of"

$\mathbb{Z}$ = "the set of integers"

Solve for $x \in [0, 2\pi)$.

2. $2 \sin x = \sqrt{3}$

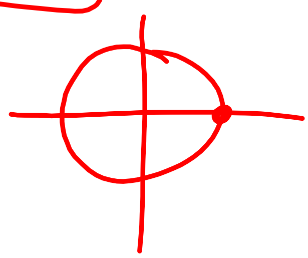
$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

4. $\cos x - 1 = 0$

$\cos x = 1$

$\boxed{x = 0}$



6. $2\sin x\cos x=\sqrt{3}\sin x$

$2\sin x\cos x-\sqrt{3}\sin x=0$

$\sin x\,(2\cos x-\sqrt{3})=0$

$\sin x=0$ or $2\cos x-\sqrt{3}=0$

$\boxed{x=0,\pi}$

$2\cos x=\sqrt{3}$

$\cos x=\frac{\sqrt{3}}{2}$

$\boxed{x=\frac{\pi}{6},\frac{11\pi}{6}}$

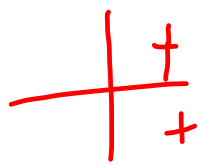
$\frac{x^2}{x}=\frac{x}{x}$

$x=1$

$x^2-x=0$

$x(x-1)=0$

$x=0,\ x=1$



Algebra Review

$(x-2)(x-3)(x-4)=0$

$x-2=0,\ x-3=0,\ x-4=0$

$x=2,\ x=3,\ x=4$

The **Zero Product Property** states:
If $AB=0$, then $A=0$ or $B=0$.

$x^2 = 9$

$x = \pm 3$

The **Square Root Theorem** states:

If $[f(x)]^2 = c$, then $f(x) = \pm\sqrt{c}$

8. $\cos^2 x - 1 = 0 \xrightarrow{\text{OR}} (\cos x - 1)(\cos x + 1) = 0$

$\cos^2 x = 1$

$\cos x = \pm 1$

$\cos x = 1$; $\cos x = -1$

$\boxed{x = 0 \qquad x = \pi}$

Let $u = \sec x$

10. $\sec^2 x + \sqrt{3}\sec x - \sqrt{2}\sec x - \sqrt{6} = 0$

$\underbrace{u^2 + \sqrt{3}u}\ \underbrace{-\sqrt{2}u - \sqrt{6}} = 0$

$u(u+\sqrt{3}) - \sqrt{2}(u+\sqrt{3}) = 0$

$(u+\sqrt{3})(u-\sqrt{2}) = 0$

$(\sec x+\sqrt{3})(\sec x-\sqrt{2}) = 0$

$\sec x = -\sqrt{3}$ $\qquad$ $\sec x = \sqrt{2}$

$x = \sec^{-1}(-\sqrt{3})$ $\qquad$ $\boxed{x = \pi/4,\ 7\pi/4}$

14. $2\cos^2 x + 1 = -3\cos x$

$2\cos^2 x + 3\cos x + 1 = 0$

Let $u = \cos x$

$2u^2 + 3u + 1 = 0$

$(2u+1)(u+1) = 0$

$(2\cos x+1)(\cos x+1) = 0$

$2\cos x + 1 = 0$ $\qquad$ $\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$ $\qquad$ $\cos x = -1$

$x = 2\pi/3,\ 4\pi/3$ $\qquad$ $\boxed{x = \pi}$

18. $4\cos^3 x = 3\cos x$

$$4\cos^3 x - 3\cos x = 0$$

$$\cos x(4\cos^2 x - 3) = 0$$

$$\cos x = 0 \qquad 4\cos^2 x - 3 = 0$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \cos^2 x = \frac{3}{4}$$

$$\cos x = \pm\frac{\sqrt{3}}{2}$$

$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3}\tan x$

$$\underbrace{\tan^2 x + \tan x}\ \underbrace{-\sqrt{3}\tan x - \sqrt{3}} = 0$$

$$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$$

$$(\tan x + 1)(\tan x - \sqrt{3}) = 0$$

$$\tan x = -1 \qquad \tan x = \frac{\sqrt{3}}{1}$$

$$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4} \qquad x = \frac{\pi}{3}, \frac{4\pi}{3}}$$

22. $\cos^4 x = \cos^2 x$

$\cos^4 x - \cos^2 x = 0$

$\cos^2 x(\cos^2 x - 1) = 0$

$\cos^2 x = 0$ $\qquad$ $\cos^2 x = 1$

$\cos x = 0$ $\qquad$ $\cos x = \pm 1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$ $\qquad$ $\boxed{x = 0, \pi}$

New Directions: <u>Find ALL the solutions</u> (not just in $[0,2\pi)$)

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$

$\sec 3x = \frac{2\sqrt{3}}{3}$

$\sec(3x) = \frac{2}{\sqrt{3}}$

$3x = \pi/6 + 2\pi k; 3x = 11\pi/6 + 2\pi k$

$\boxed{x = \pi/18 + \frac{2\pi}{3}k \; ; \; x = \frac{11\pi}{18} + \frac{2\pi}{3}k}$

68. $\cos\left(2x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2}$

$= -\frac{1}{\sqrt{2}}$

$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k$, $2x - \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$		$2x = \frac{3\pi}{2} + 2\pi k$

$\boxed{x = \frac{\pi}{2} + \pi k}$		$\boxed{x = \frac{3\pi}{4} + \pi k}$

Homework for Test #3:

Homework #6 (submitted Wed. 01/15)
- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

Homework #7 (due Wednesday?)
- **6.5 #1-24 all; 25-55 odd**
- **6.6 #1-21 odd - finding solutions between 0 and 2pi**
- **6.6 #61-69 odd - finding all possible solutions (+2pi*k)**

Test #3 - Thursday 01/23?