Solve for $x \in [0, 2\pi)$.

$\sin x - \cos x = 1$

$(\sin x - \cos x)^2 = 1^2$

$\sin^2 x - 2\sin x\cos x + \cos^2 x = 1$

$(\sin^2 x + \cos^2 x) - 2\sin x \cos x = 1$

$1 - \sin 2x = 1$

$-\sin 2x = 0$

$\sin \boxed{2x} = 0$

$2x = 0, \pi, 2\pi, 3\pi$

$x = \cancel{0}, \boxed{\frac{\pi}{2}}, \boxed{\pi}, \cancel{\frac{3\pi}{2}}$

* raising both sides of an equation to an even power may introduce extraneous solutions!

$(a-b)^2 = a^2 - 2ab + b^2$

$0 \le x < 2\pi$

$0 \le 2x < 4\pi$

Check:

$\sin 0 - \cos 0 = 0 - 1 = -1$

$\sin\frac{\pi}{2} - \cos\frac{\pi}{2} = 1 - 0 = 1$

$\sin\pi - \cos\pi = 0 - (-1) = 1$

$\sin\frac{3\pi}{2} - \cos\frac{3\pi}{2} = -1 - 0 = -1$

$$\cos(4x) = \frac{1}{\sqrt{2}}$$

look for solutions 4 times around the unit circle

$0 \le x < 2\pi$

$0 \le 4x < 8\pi$

$2\pi \cdot \frac{4}{4} = \frac{8\pi}{4}$

$4x = \frac{\pi}{4}, \frac{7\pi}{4}, \frac{9\pi}{4}, \frac{15\pi}{4}, \frac{17\pi}{4}, \frac{23\pi}{4}, \frac{25\pi}{4}, \frac{31\pi}{4}$

$x = \frac{\pi}{16}, \frac{7\pi}{16}, \frac{9\pi}{16}, \frac{15\pi}{16}, \frac{17\pi}{16}, \frac{23\pi}{16}, \frac{25\pi}{16}, \frac{31\pi}{16}$

$\tan(5x) = 0$

$0 \le x < 2\pi$

$0 \le 5x < 10\pi$

$5x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, 8\pi, 9\pi$

$$\boxed{x = 0, \frac{\pi}{5}, \frac{2\pi}{5}, \frac{3\pi}{5}, \frac{4\pi}{5}, \pi, \frac{6\pi}{5}, \frac{7\pi}{5}, \frac{8\pi}{5}, \frac{9\pi}{5}}$$

$x \in [0, 2\pi)$

72. $\cos 2x = 2\cos x - 1$

$2\cos^2 x - 1 = 2\cos x - 1$

$+1 \qquad\qquad +1$

$2\cos^2 x = 2\cos x$

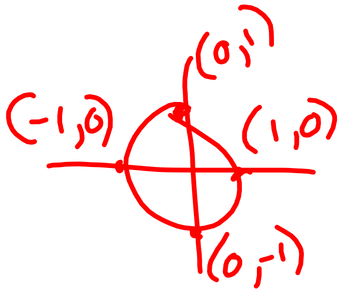
$2\cos^2 x - 2\cos x = 0$

$2\cos x(\cos x - 1) = 0$

$2\cos x = 0$; $\cos x - 1 = 0$

$\cos x = 0$ $\qquad$ $\cos x = 1$

$$\boxed{x = \pi/2, 3\pi/2 \qquad x = 0}$$



$x \in [0, 2\pi)$

74. $\sin 4x - \cos 2x = 0$

$\sin 2\theta = 2\sin\theta\cos\theta$

$\sin 2(2x) - \cos 2x = 0$

$2\sin 2x \cos 2x - \cos 2x = 0$

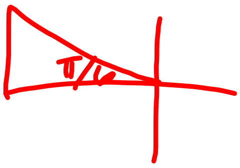
$\cos 2x (2\sin 2x - 1) = 0$

$\cos 2x = 0$

$2x = \pi/2,\ 3\pi/2,\ 5\pi/2,\ 7\pi/2$

$\sin 2x = \frac{1}{2}$

$\frac{6\pi}{6} = \pi$



$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$

$x = \pi/4,\ 3\pi/4,\ 5\pi/4,\ 7\pi/4,\ \pi/12,\ \frac{5\pi}{12},\ \frac{13\pi}{12},\ \frac{17\pi}{12}$

$x \in [0, 2\pi)$

78. $\underbrace{\cos 2x}_{a} \underbrace{\cos x}_{b} - \underbrace{\sin 2x}_{a} \underbrace{\sin x}_{b} = 0$

$\cos(2x + x) = 0$

$\cos 3x = 0$

$3x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2}$

$x = \pi/6,\ \pi/2,\ \frac{5\pi}{6},\ \frac{7\pi}{6},\ \frac{3\pi}{2},\ \frac{11\pi}{6}$

$x \in [0, 2\pi)$

82. $\cos 3x + \cos x = 0$

$\cos(2x + x) + \cos x = 0$

$\cos 2x \cos x - \sin 2x \sin x + \cos x = 0$

$(1 - 2\sin^2 x)\cos x - (2\sin x \cos x)\sin x + \cos x = 0$

$\cos x - 2\sin^2 x \cos x - 2\sin^2 x \cos x + \cos x = 0$

$2\cos x - 4\sin^2 x \cos x = 0$

$\cos x (2 - 4\sin^2 x) = 0$

$\cos x = 0$

$\boxed{x = \pi/2, 3\pi/2}$

$2 - 4\sin^2 x = 0$

$-4\sin^2 x = -2$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm \frac{1}{\sqrt{2}}$

$\boxed{x = \pi/4, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}}$

$x \in [0, 2\pi)$

84. $2 \sin x \cos x - 2\sqrt{2} \sin x - \sqrt{3} \cos x + \sqrt{6} = 0$

$2\sin x(\cos x - \sqrt{2}) - \sqrt{3}(\cos x - \sqrt{2}) = 0$

$(\cos x - \sqrt{2})(2\sin x - \sqrt{3}) = 0$

$\cancel{\cos x = \frac{\sqrt{2}}{1}}$

$\sin x = \frac{\sqrt{3}}{2}$

$\boxed{x = \pi/3, 2\pi/3}$

$x \in [0, 2\pi)$

76. $\tan\frac{x}{2} = 1 - \cos x$

$$\frac{1-\cos x}{\sin x} = 1 - \cos x$$

$$\frac{1-\cos x}{\sin x} - (1-\cos x) = 0$$

$$\frac{1-\cos x}{\sin x} - \frac{\sin x}{\sin x} + \frac{\sin x\cos x}{\sin x} = 0$$

$$\frac{1-\cos x - \sin x + \sin x\cos x}{\sin x} = 0$$

$$\frac{1-\cos x - \sin x(1-\cos x)}{\sin x} = 0$$

$$\frac{(1-\cos x)(1-\sin x)}{\sin x} = 0$$

$1-\cos x = 0$, $1-\sin x = 0$

$\cos x = 1$, $\sin x = 1$

$$\boxed{x = 0, \; x = \frac{\pi}{2}}$$

Homework for Test #3:

Homework #6 (submitted Wed. 01/15)
- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

Homework #7 (due Wednesday 01/22)
- 6.5 #1-24 all; 25-55 odd
- 6.6 #1-21 odd - finding solutions between 0 and 2pi
- 6.6 #61-69 odd - finding all possible solutions (+2pi*k)
- **6.6 #71-83 odd;**
- **Examples #3,4,7,8 from solving equations handout**
- **Test 3 Practice Problems (counts as take-home quiz!)**

Test #3 - Thursday 01/23