Homework for Test #3:

Homework #6 (submitted Wed. 01/15)

- 6.1 #1-69 odd (proofs)
- 6.2 #1-41 odd
- 6.3 #1-24 all; 30-36 all; 49-93 odd

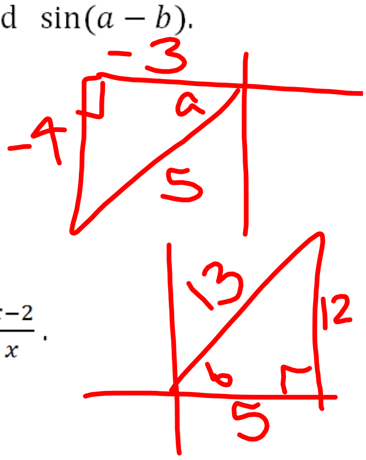
Homework #7 (due Wednesday 01/22)

- 6.5 #1-24 all; 25-55 odd
- 6.6 #1-21 odd - finding solutions between 0 and 2pi
- 6.6 #61-69 odd - finding all possible solutions (+2pi*k)
- **6.6 #71-83 odd;**
- **Examples #3,4,7,8 from solving equations handout**
- **Test 3 Practice Problems (counts as take-home quiz!)**

Test #3 - Thursday 01/23

1. Given that $\sin a = -\frac{4}{5}$, $a \in QIII$, and $\tan b = \frac{12}{5}$, $b \in QI$, find $\sin(a-b)$.

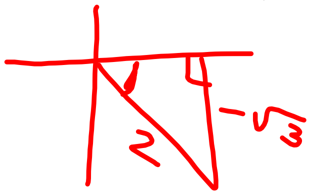
$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$
$$= \left(-\frac{4}{5}\right)\left(\frac{5}{13}\right) - \left(\frac{-3}{5}\right)\left(\frac{12}{13}\right)$$
$$= \frac{-20}{65} + \frac{36}{65} = \boxed{\frac{16}{65}}$$



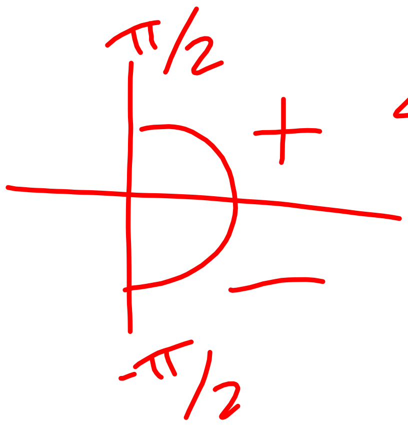
2. Simplify and express as a single trigonometric function $\frac{\csc^2 x - 2}{\csc^2 x}$.

$$\frac{\csc^2 x}{\csc^2 x} - \frac{2}{\csc^2 x} = 1 - 2\sin^2 x$$
$$= \boxed{\cos 2x}$$

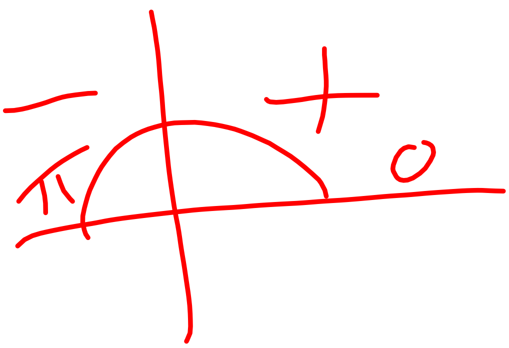
3. Evaluate $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$. Give the answer in radians.



$$\boxed{-\frac{\pi}{3}}$$



sin, csc, tan



cos, sec, cot

1. Use the half-angle identity to evaluate $\tan\frac{7\pi}{12}$ exactly.

$$\tan\frac{\theta}{2} = \frac{1-\cos\theta}{\sin\theta}$$

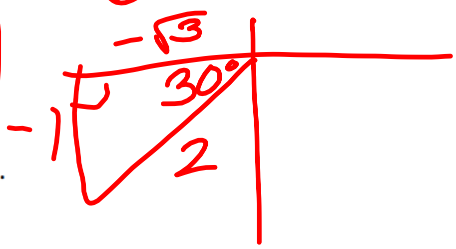
$$\frac{7\pi}{12} = \frac{\theta}{2}$$

$$\frac{7\pi}{6} = \theta$$

$$\tan\frac{7\pi}{12} = \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{-\frac{1}{2}} = \frac{1+\frac{\sqrt{3}}{2}}{-\frac{1}{2}} =$$

$$= \left(1+\frac{\sqrt{3}}{2}\right)\left(-\frac{2}{1}\right) = \boxed{-2-\sqrt{3}}$$



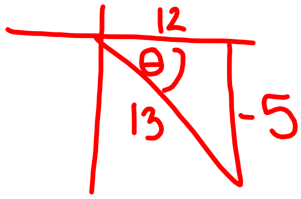
2. Find the exact value of $\cos 212° \cos 122° + \sin 212° \sin 122°$.

$$= \cos(212° - 122°)$$

$$= \cos 90°$$

$$= \boxed{0}$$

3. Find $\sin 2\theta$, $\cos 2\theta$, *and* $\tan 2\theta$ given that $\cos\theta = \frac{12}{13}$ and θ is in Quadrant IV.



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-5}{13}\right)\left(\frac{12}{13}\right) = \boxed{\frac{-120}{169}}$

$\boxed{2\theta \in QIV}$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

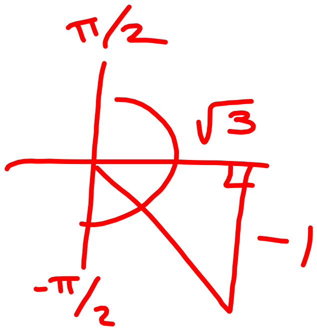
$= (\cos\theta)^2 - (\sin\theta)^2$

$= \left(\frac{12}{13}\right)^2 - \left(\frac{-5}{13}\right)^2 = \frac{144}{169} - \frac{25}{169} = \boxed{\frac{119}{169}}$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \frac{-120/169}{119/169} = \boxed{\frac{-120}{119}}$

4. Given $\sin\alpha = \frac{12}{13}$, α is in Quadrant II, $\cos\beta = -\frac{4}{5}$, and β is in Quadrant III, find $\sin(\alpha + \beta)$.

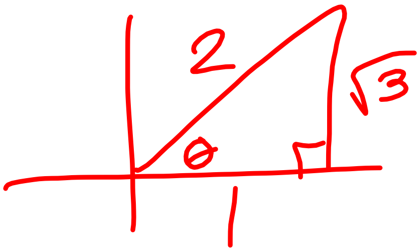
5. Find $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ exactly in radians.



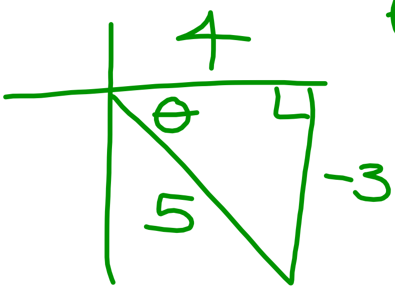
$$\boxed{-\frac{\pi}{6}}$$

6. Evaluate $\cos\left(\underbrace{\csc^{-1}\frac{2}{\sqrt{3}}}_{\theta}\right)$

$$=\boxed{\frac{1}{2}}$$



$$\sec\left(\underbrace{\sin^{-1}\left(-\frac{3}{5}\right)}_{\theta}\right)=\boxed{\frac{5}{4}}$$



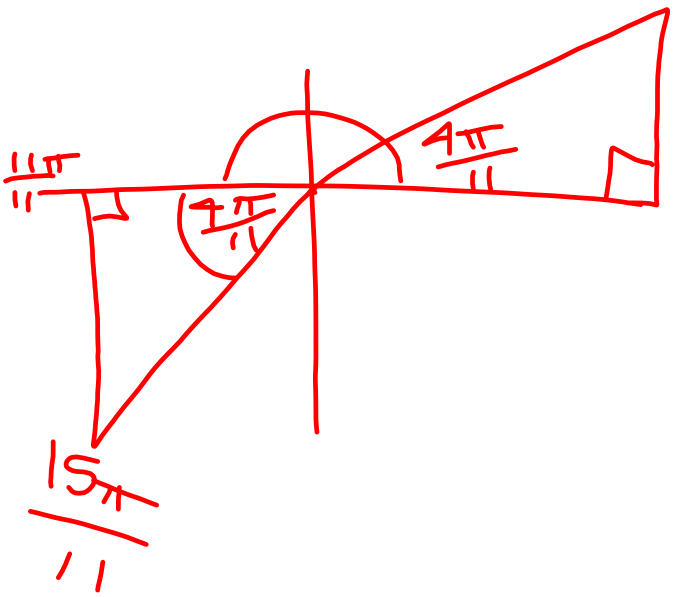
$$\sin^{-1}\left(\sin\frac{2\pi}{7}\right) = \boxed{\frac{2\pi}{7}}$$

f & g are inverses if

$(f\circ g)(x) = x$ & $(g\circ f)(x) = x$

$\sqrt[3]{31^3} = 31$ $\left(\sqrt[3]{31}\right)^3 = 31$

$$\cot^{-1}\left(\cot\left(\frac{15\pi}{11}\right)\right) = \boxed{\frac{4\pi}{11}}$$



7. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $\sin^2 x - \frac{1}{4} = 0$

$$\sin^2 x = \frac{1}{4}$$

$$\sin x = \pm \frac{1}{2}$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

8. Find all solutions (in radians) in the interval $0 \le x < 2\pi$. $2\sin^3 x = \sin x$

$$2\sin^3 x - \sin x = 0$$

$$\sin x(2\sin^2 x - 1) = 0$$

$$\sin x = 0 \qquad 2\sin^2 x - 1 = 0$$

$$\sin^2 x = \frac{1}{2}$$

$$x = 0, \pi \qquad \sin x = \pm\frac{1}{\sqrt{2}}$$

$$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$$

9. Prove the identity.

$$\frac{1+\cos^2 x}{\sin^2 x} = 2\csc^2 x - 1$$

$$\frac{\sin^2 x + \cos^2 x = 1}{\sin^2 x}$$

$1 + \cot^2 x = \csc^2 x$

$-1 \quad -1$

$\cot^2 x = \csc^2 x - 1$

$$LHS = \frac{1+\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} =$$

$$= \csc^2 x + \cot^2 x = \csc^2 x + (\csc^2 x - 1)$$

$$= 2\csc^2 x - 1 = RHS \checkmark$$

10. Prove the identity. $\csc x - \cos x \cot x = \sin x$

$$LHS = \csc x - \cos x \cot x$$

$$= \frac{1}{\sin x} - \frac{\cos x}{1} \cdot \frac{\cos x}{\sin x}$$

$$= \frac{1-\cos^2 x}{\sin x}$$

$$= \frac{\sin^2 x}{\sin x}$$

$$= \sin x = RHS \checkmark$$

Bonus (10 points): Find all solutions (in radians) in the interval $0 \leq x < 2\pi$.

$$\sin 3x + \sin x - \sin 2x = 0$$