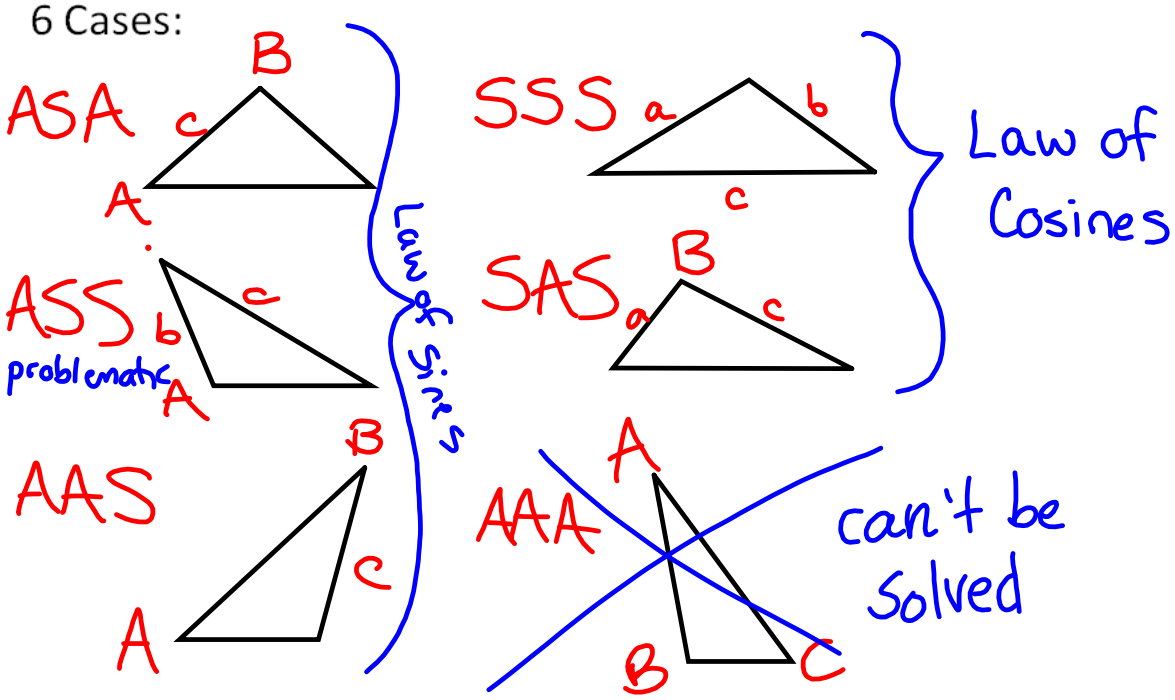
7.1 The Law of Sines

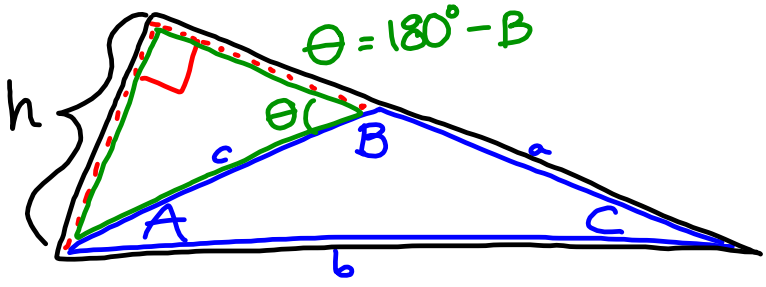
How do we solve oblique (not right) triangles?

We need to know three things about a triangle in order to solve it.

6 Cases:



Derivaon of the Law of Sines



$$\frac{b\sin C}{\sin B\sin C} = \frac{c\sin B}{\sin B\sin C}$$

$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin C = \frac{h}{b}$$

$$h = b\cdot\sin C$$

$$\frac{b\sin C}{bc} = \frac{c\sin B}{bc}$$

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin(180°-B) = \frac{h}{c}$$

$$\sin 180°\cos B - \cos 180°\sin B = \frac{h}{c}$$

$$0\cdot\cos B - (-1)\cdot\sin B = \frac{h}{c}$$

$$\sin B = \frac{h}{c}$$

$$h = c\cdot\sin B$$

The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

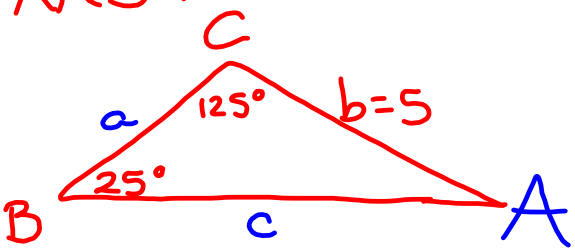
or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

7.1

2. $B = 25°$, $C = 125°$, $b = 5$

AAS ⇒ Law of Sines



$A = 180° - 125° - 25°$

$A = 30°$

$\frac{c}{\sin C} = \frac{b}{\sin B}$

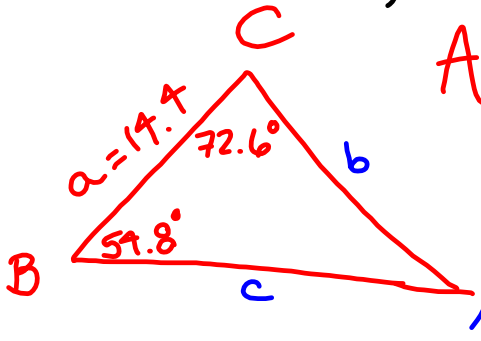
$\frac{c}{\sin 125°} = \frac{5}{\sin 25°}$

$c = \frac{5 \cdot \sin 125°}{\sin 25°} \approx 9.7 = c$

$\frac{a}{\sin 30°} = \frac{5}{\sin 25°}$

$a = \frac{5 \sin 30°}{\sin 25°} \approx 5.9 = a$

8. $B = 54.8^\circ$, $C = 72.6^\circ$, $a = 14.4$



ASA ⇒ Law of Sines

$A = 180^\circ - 54.8^\circ - 72.6^\circ$

$A = 52.6^\circ$

$\frac{b}{\sin B} = \frac{a}{\sin A}$

$b = \frac{14.4 \cdot \sin 54.8^\circ}{\sin 52.6^\circ}$

$b \approx 14.8$

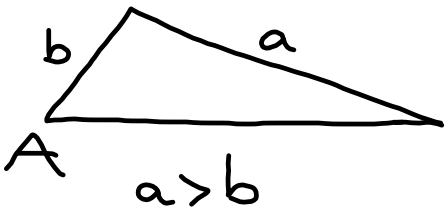
$\frac{c}{\sin C} = \frac{a}{\sin A}$

$c = \frac{14.4 \cdot \sin 72.6^\circ}{\sin 52.6^\circ}$

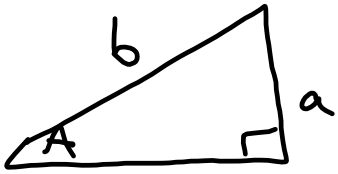
$c \approx 17.3$

ASS, The Problematic Triangle

one solution:



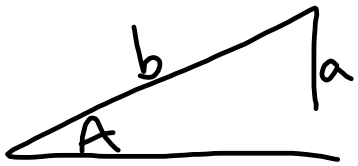
$a > b$



$a < b$

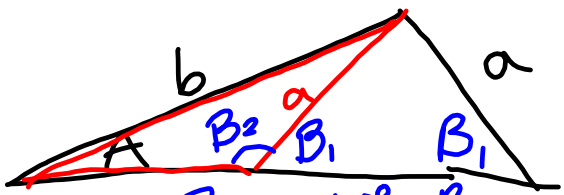
$B = 90^\circ$

no solutions:



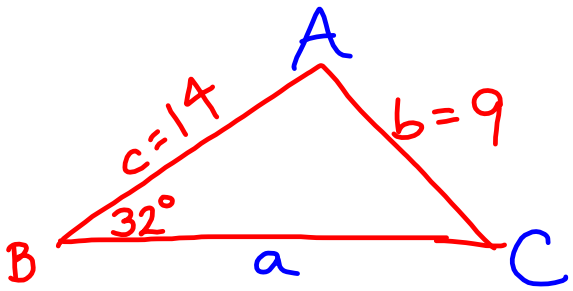
$a < b$

two solutions: $a < b$



$B_2 = 180^\circ - B_1$

14. $B = 32°, c = 14, b = 9$



ASS ⇒ Law of Sines (problematic!)

$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\sin C = \frac{14 \cdot \sin 32°}{9}$$

$$C = \sin^{-1}\left(\frac{14\sin 32°}{9}\right)$$

$$\approx 55.5°$$

Case 1

$C_1 = 55.5°$

$A_1 = 180° - 32° - 55.5°$

$A_1 = 92.5°$

$$\frac{a_1}{\sin 92.5°} = \frac{9}{\sin 32°}$$

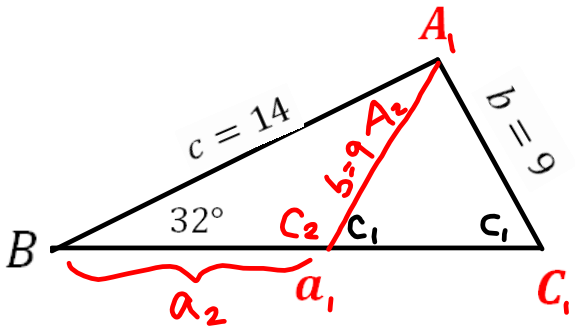
$$a_1 = \frac{9\sin 92.5°}{\sin 32°} \approx 17.0 = a_1$$

7.1 The Law of Sines, connued

ASS – Problematic Triangle

14. $B = 32°, c = 14, b = 9$

Case 1: $C \approx 55.5°, A \approx 92.5°, a \approx 17$



$C_2 = 180° - C_1$

$= 180° - 55.5°$

$C_2 = 124.5°$

$A_2 = 180° - 32° - 124.5°$

$A_2 = 23.5°$

$$\frac{a_2}{\sin 23.5°} = \frac{9}{\sin 32°}$$

$$a_2 = \frac{9\sin 23.5°}{\sin 32°}$$

$a_2 \approx 6.8$

Homework:

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines

- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines