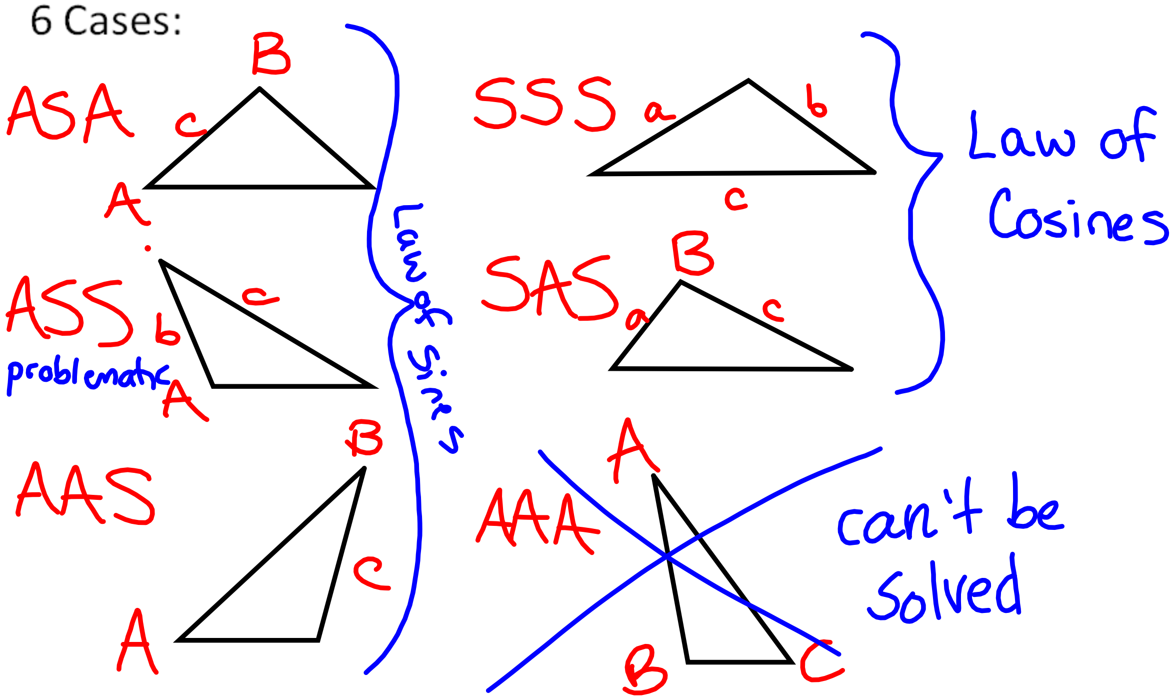
7.1 The Law of Sines

How do we solve oblique (not right) triangles?

We need to know three things about a triangle in order to solve it.

6 Cases:



The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

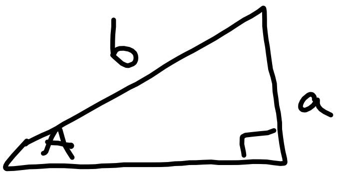
or

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

ASS, The Problematic Triangle

one solution:

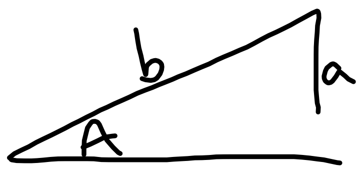
$a > b$



$a < b$

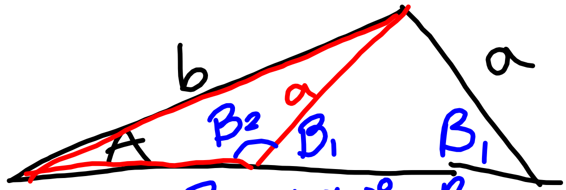
$B = 90^\circ$

no solutions:



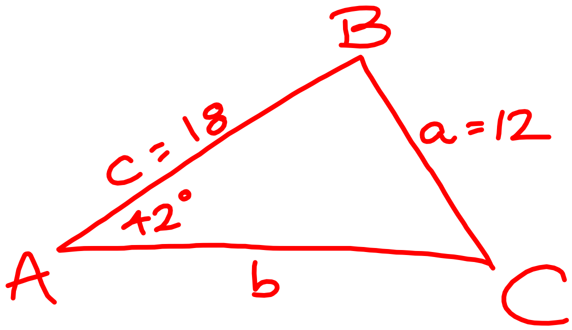
$a < b$

two solutions: $a < b$



$B_2 = 180^\circ - B_1$

16. $A = 42^\circ, a = 12, c = 18$



$\boxed{\text{no } \triangle \text{ exists}}$

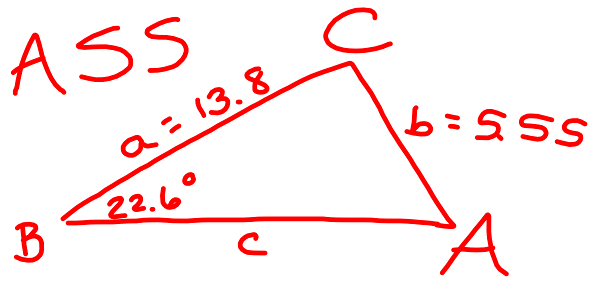
$$\frac{\sin C}{18} = \frac{\sin 42^\circ}{12}$$

$$\sin C = \frac{18 \sin 42^\circ}{12} \approx 1.00369 > 1$$

$$C = \sin^{-1}\left(\frac{18 \sin 42^\circ}{12}\right) = \text{undefined}$$

$18.\ B = 22.6°,\ b = 5.55,\ a = 13.8$

ASS



$$\frac{\sin A}{13.8} = \frac{\sin 22.6°}{5.55}$$

$$\sin A = \frac{13.8 \sin 22.6°}{5.55}$$

$$A = \sin^{-1}\left(\frac{13.8 \sin 22.6°}{5.55}\right)$$

$$\boxed{A_1 \approx 72.9°}$$

CASE 1

$$C = 180° - 22.6° - 72.9°$$

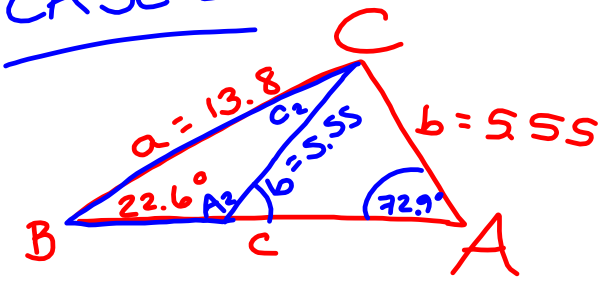
$$\boxed{C_1 = 84.5°}$$

$$\frac{c}{\sin 84.5°} = \frac{5.55}{\sin 22.6°}$$

$$c = \frac{5.55 \sin 84.5°}{\sin 22.6°}$$

$$\boxed{c_1 = 14.4}$$

CASE 2



$$A_2 = 180° - A_1$$

$$= 180° - 72.9°$$

$$\boxed{A_2 = 107.1°}$$

$$C_2 = 180° - B - A_2$$

$$= 180° - 22.6° - 107.1°$$

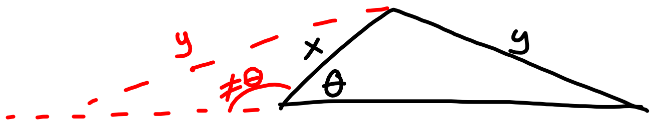
$$\boxed{C_2 = 50.3°}$$

$$\frac{c_2}{\sin C_2} = \frac{5.55}{\sin 22.6°}$$

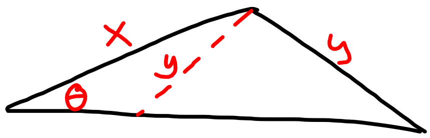
$$c_2 = \frac{5.55 \sin(50.3°)}{\sin 22.6°}$$

$$\boxed{c_2 = 11.1}$$

Why does this ASS triangle have only one solution?



The measure of θ and the lengths of x & y are fixed. If we try to reposition y, the measure of θ changes, unlike in the 2-solution case:



7.2 - The Law of Cosines

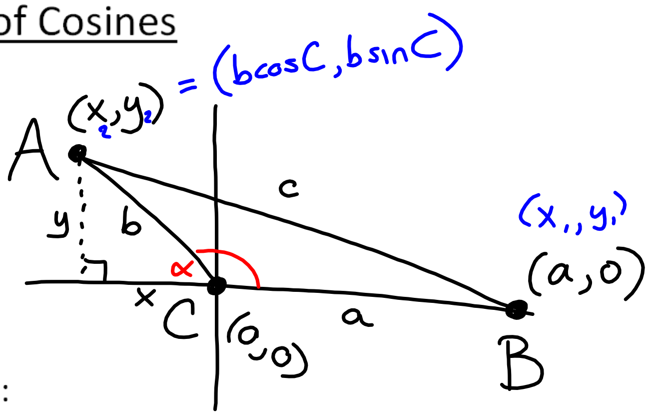
Derivation:

$\cos C = \frac{x}{b}$

$x = b\cos C$

$\sin C = \frac{y}{b}$

$y = b\sin C$



Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$c^2 = (b\cos C - a)^2 + (b\sin C - 0)^2$$

$$c^2 = b^2\cos^2 C - 2ab\cos C + a^2 + b^2\sin^2 C$$

$$c^2 = a^2 + b^2\underbrace{(\sin^2 C + \cos^2 C)}_{=1} - 2ab\cos C$$

$$\boxed{c^2 = a^2 + b^2 - 2ab\cos C}$$

The Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos B$$

$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

7.2

16. $a = 60, b = 88, c = 120.$ $B = ?$

SSS ⇒ Law of Cosines

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$2ac\cos B = a^2 + c^2 - b^2$$

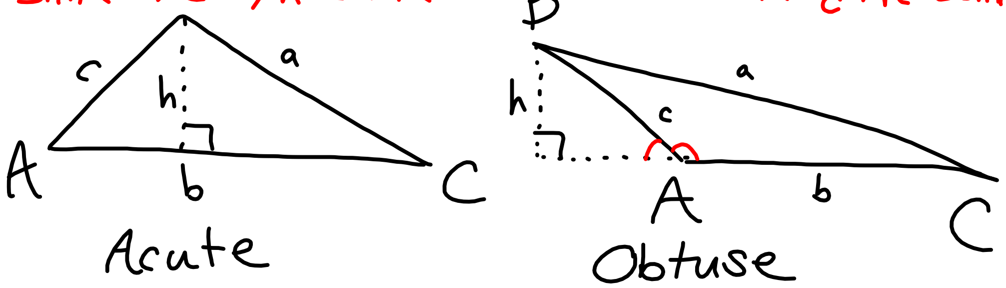
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$B = \cos^{-1}\left(\frac{a^2+c^2-b^2}{2ac}\right) = \cos^{-1}\left(\frac{60^2+120^2-88^2}{2(60)(120)}\right) = \boxed{44.6^\circ}$$

cos⁻¹((60^2+120^2-88^2)/(2*60*120))

7.1/7.2 Area of a Triangle

$\sin A = h/c \Rightarrow h = c\sin A$ $\sin A = \frac{h}{c} \Rightarrow h = c\sin A$



Area of a triangle $= \frac{1}{2} \cdot$ base $\cdot$ height

$$\text{Area} = \frac{1}{2} \cdot b \cdot c \cdot \sin A$$

$$= \frac{1}{2} ac\sin B$$

$$= \frac{1}{2} ab\sin C$$

Find the area of the triangle.

$A = 50°, b = 13\ cm, c = 6\ cm$

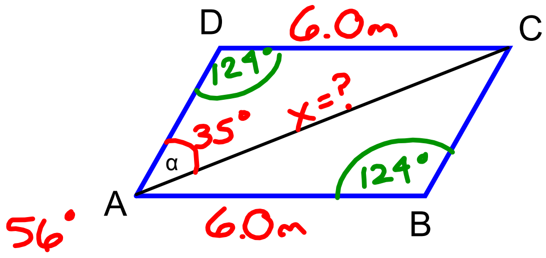
$$\text{Area} = \frac{1}{2} bc\sin A$$

$$= \frac{1}{2}(13)(6)\sin 50°$$

$$= \boxed{29.9\ cm^2}$$

7.1 #28

The longer side of a parallelogram is 6.0 meters. The measure of angle BAD is 56° and α is 35°. Find the length of the longer diagonal.



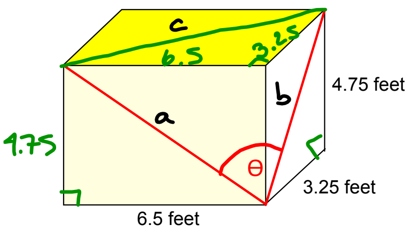
$$\frac{x}{\sin 124^\circ} = \frac{6}{\sin 35^\circ}$$

$$x = \frac{6\sin 124^\circ}{\sin 35^\circ}$$

$$= \boxed{8.7\text{ m}}$$

7.2 #41

The rectangular box in the igure measures 6.50 feet by 3.25 feet by 4.75 feet. Find the measure of the angle ϴ that is formed by the union of the diagonal shown on the front of the box and the diagonal shown on the right side of the box.



$$a = \sqrt{4.75^2 + 6.5^2}$$
$$= 8.05$$

$$b = \sqrt{4.75^2 + 3.25^2}$$
$$= 5.76$$

$$c = \sqrt{6.5^2 + 3.25^2}$$
$$= 7.27$$

$$c^2 = a^2 + b^2 - 2ab\cdot\cos\theta$$
$$2ab\cos\theta = a^2 + b^2 - c^2$$
$$\cos\theta = \frac{a^2+b^2-c^2}{2ab}$$
$$\theta = \cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right) =$$
$$= \cos^{-1}\left(\frac{8.05^2+5.76^2-7.27^2}{2(8.05)(5.76)}\right) =$$
$$= \boxed{60.88^\circ}$$

Homework:

- 7.1 #1-21 odd — solving triangles with Law of Sines
- 7.1 #29,30,33,34,35 — word problems with Law of Sines

- 7.2 #9-19 odd — solving triangles with Law of Cosines
- 7.2 #25-29odd; — area
- 7.2 #38,43,46,47,48 — word problems with Law of Cosines

Expect a quiz on Law of Sines/Cosines on Thursday

Homework due Friday

Test #4 Canceled

Wed, Thurs: continue with 7.1/7.2

Fri, Mon: review for Final Exam