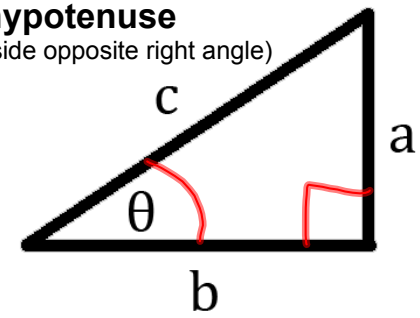
5.1 Trigonometric Functions of Acute Angles

An acute angle is an angle between 0° & 90° .

A right triangle is a triangle with a 90° angle.



hypotenuse
(side opposite right angle)

opposite side
(side opposite angle of interest)

adjacent side
(side adjacent to angle of interest)

θ theta

α alpha

β beta

γ gamma

The six basic trigonometric functions are ratios of sides of a right triangle.

sine $\sin\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ hypotenuse} = \frac{opp}{hyp}$

cosine $\cos\theta = \frac{length\ of\ side\ adjacent\ to\ \theta}{length\ of\ hypotenuse} = \frac{adj}{hyp}$

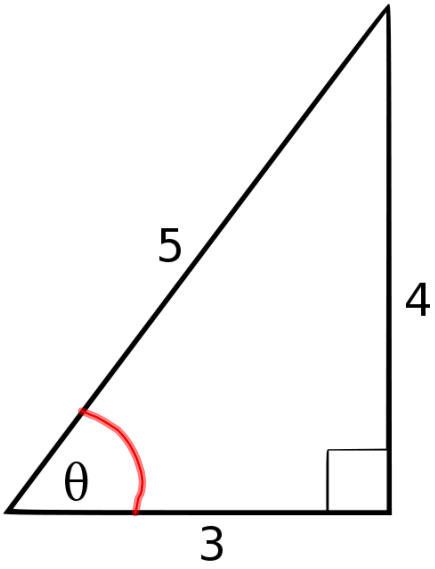
tangent $\tan\theta = \frac{length\ of\ side\ opposite\ \theta}{length\ of\ side\ adjacent\ to\ \theta} = \frac{opp}{adj}$

SohCahToa

secant $\sec\theta = \frac{hyp}{adj} = \frac{1}{\cos\theta}$

cosecant $\csc\theta = \frac{hyp}{opp} = \frac{1}{\sin\theta}$

cotangent $\cot\theta = \frac{adj}{opp} = \frac{1}{\tan\theta}$



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

$\tan\theta = \frac{4}{3}$

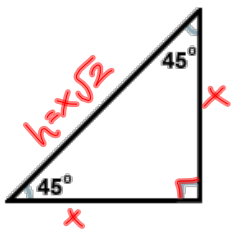
$\sec\theta = \frac{5}{3}$

$\csc\theta = \frac{5}{4}$

$\cot\theta = \frac{3}{4}$

Two special right triangles:

Isosceles Right Triangle aka 45-45-90



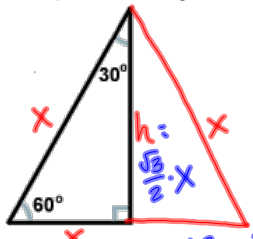
$x^2+x^2=h^2$

$2x^2=h^2$

$\sqrt{2x^2}=h$

$x\sqrt{2}=h$

Half of an equilateral triangle aka 30-60-90



$\left(\frac{x}{2}\right)^2+h^2=x^2$

$\frac{x^2}{4}+h^2=x^2$

$h^2=\frac{4x}{4}-\frac{x^2}{4}$

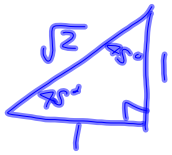
$h^2=\frac{3x}{4}$

$h=\frac{\sqrt{3}}{2}x$

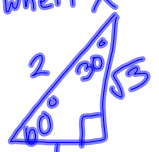
Pythagorean Theorem:

$a^2+b^2=c^2$, a,b - legs; c - hypotenuse

when x = 1



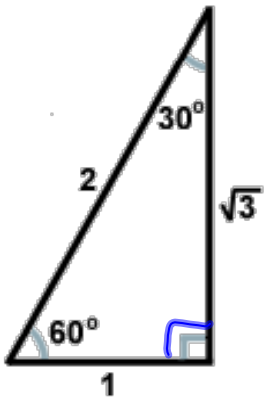
when x = 2



$30^\circ < 60^\circ < 90^\circ$

$\sqrt{1} < \sqrt{3} < \sqrt{4}$

$1 < \sqrt{3} < 2$



30° & 60° are complimentary

$\sin 30° = \frac{1}{2}$ $\quad$ $\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$ $\quad$ $\cos 60° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ $\quad$ $\tan 60° = \sqrt{3}$

$\sec 30° = \frac{2}{\sqrt{3}}$ $\quad$ $\sec 60° = 2$

$\csc 30° = 2$ $\quad$ $\csc 60° = \frac{2}{\sqrt{3}}$

$\cot 30° = \sqrt{3}$ $\quad$ $\cot 60° = \frac{1}{\sqrt{3}}$

Cofunctions

The function of an angle is equal to the cofunction of its complement.

The complement of an angle θ is equal to $90° - \theta$.

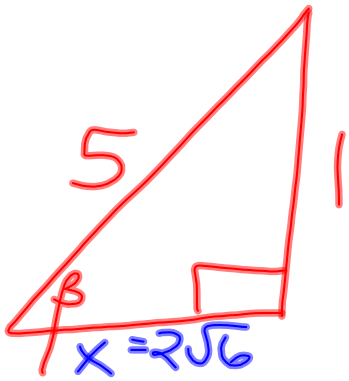
Examples:

$\cos(20°) = \sin(90° - 20°) = \sin 70°$

$\csc(89°) = \sec 1°$

$\tan(7°) = \cot 83°$

Given that $\csc\beta = 5$, find the other trigonometric function values of β.



$x^2+1^2=5^2$

$x^2=25-1$

$x^2=24=4\cdot6$

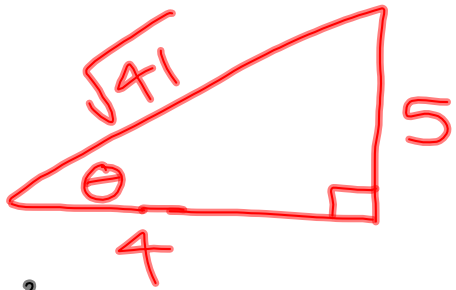
$x=2\sqrt{6}$

$\sin\beta=\frac{1}{5}$ $\sec\beta=\frac{5}{2\sqrt{6}}$

$\cos\beta=\frac{2\sqrt{6}}{5}$ $\cot\beta=2\sqrt{6}$

$\tan\beta=\frac{1}{2\sqrt{6}}$

Given that $\cot\theta = \frac{4}{5}$, find the other trigonometric function values of θ.



$h^2=4^2+5^2$

$h^2=16+25$

$h^2=41$

$h=\sqrt{41}$

$\sec\theta=\frac{\sqrt{41}}{4}$

$\sin\theta=\frac{5}{\sqrt{41}}$

$\tan\theta=\frac{5}{4}$

Homework:

5.1#1-15odd; 17-28 all