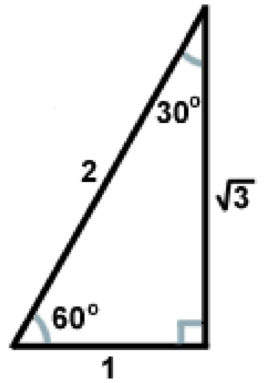
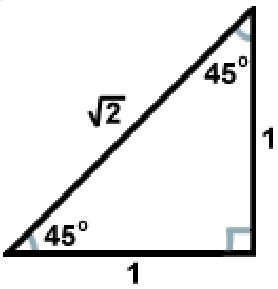
Review:

An acute angle is between 0° & 90°



The function value of an angle is equal to the cofunction value of the complement of that angle.



$\sin 45° = 1/\sqrt{2}$ $\quad$ $\sec 60° = 2$

$\cos 30° = \sqrt{3}/2$ $\quad$ $\csc 45° = \sqrt{2}$

$\tan 45° = 1$ $\quad$ $\cot 30° = \sqrt{3}$

5.2 #21 the sand dune problem:

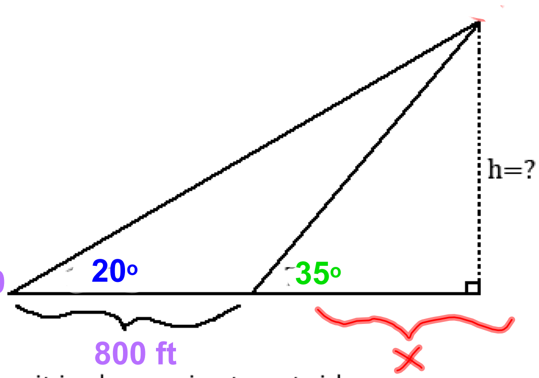
$\tan 35° = \frac{h}{x}$ $\quad$ $\tan 20° = \frac{h}{x+800}$

$x \tan 35° = h$

$x = \frac{h}{\tan 35°}$

Substitution yields:

$$\tan 20° = \frac{h}{\frac{h}{\tan 35°} + 800}$$



When confronted by an equation involving fractions, it is always nice to get rid of the fractions by multiplying both sides by the least common denominator.

$$\left(\frac{h}{\tan 35°} + 800\right) \cdot \tan 20° = \frac{h}{\cancel{\frac{h}{\tan 35°} + 800}} \cdot \cancel{\left(\frac{h}{\tan 35°} + 800\right)}$$

Distribution yields: $h \frac{\tan 20°}{\tan 35°} + 800 \tan 20° = h$

When more than one instance of your variable appears, try to get all instances of the variable on one side and everything else on the other side.

$$800 \tan 20° = h - \frac{h \tan 20°}{\tan 35°}$$

To get h by itself, factor and then divide.

$$800 \tan 20° = h\left(1 - \frac{\tan 20°}{\tan 35°}\right)$$

$$\boxed{h = \frac{800 \tan 20°}{1 - \frac{\tan 20°}{\tan 35°}} \approx 606 \text{ ft}}$$

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$, $\sin x = \frac{1}{\csc x}$, $\sec x = \frac{1}{\cos x}$, $\cos x = \frac{1}{\sec x}$, $\cot x = \frac{1}{\tan x}$, $\tan x = \frac{1}{\cot x}$

Cofunction Identities:

$\sin(90° - \theta) = \cos\theta$, $\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$, $\cot(90° - \theta) = \tan\theta$
$\csc(90° - \theta) = \sec\theta$, $\sec(90° - \theta) = \csc\theta$

Ratio Identities:

$$\frac{\sin\theta}{\cos\theta} = \frac{\left(\frac{opp}{hyp}\right)}{\left(\frac{adj}{hyp}\right)} = \frac{opp}{\cancel{hyp}} \cdot \frac{\cancel{hyp}}{adj} = \frac{opp}{adj} = \tan\theta$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \; ; \; \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Example Problem **5.1** #98

Given that

$\sin 8° \approx 0.1392$ $\csc 8° \approx 7.1853$
$\cos 8° \approx 0.9903$ $\sec 8° \approx 1.0098$
$\tan 8° \approx 0.1405$ $\cot 8° \approx 7.1154$

find the six function vales of 82°.

82° & 8° are complements

$\cos 82° = \sin 8° = 1.1392$

$\sec 82° = \csc 8° = 7.1853$

$\cot 82° = \tan 8° = 0.1405$

Write in terms of $\sin 40°$ and/or $\cos 40°$.

$\csc 40° = \dfrac{1}{\sin 40°}$

$\csc 50° = \sec 40° = \dfrac{1}{\cos 40°}$

$\cot 40° = \dfrac{\cos 40°}{\sin 40°}$

$\tan 50° = \dfrac{\sin 50°}{\cos 50°} = \dfrac{\cos 40°}{\sin 40°}$

$\cos 50° = \sin 40°$

$\sec 50° = \dfrac{1}{\cos 50°} = \dfrac{1}{\sin 40°}$

How many different ways can you rewrite the expression using ratio, reciprocal, and/or cofunction identities?

$\sin 32° = \cos(90° - 32°) = \boxed{\cos 58°}$

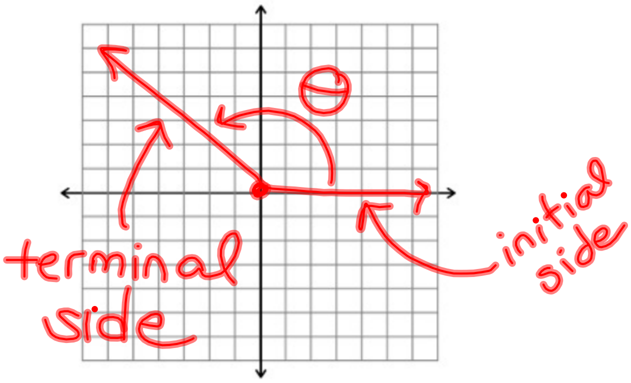
$= \boxed{\dfrac{1}{\csc 32°}} = \boxed{\dfrac{1}{\sec 58°}}$

$\tan 13° = \cot 77° = \dfrac{\sin 13°}{\cos 13°} = \dfrac{1}{\cot 13°} = \dfrac{\cos 77°}{\sin 77°}$

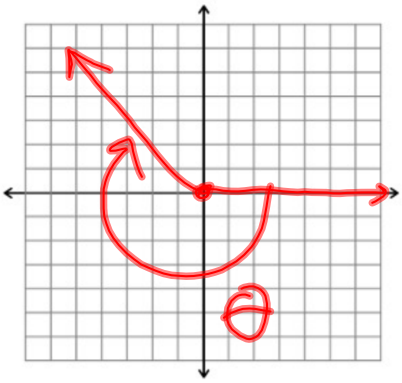
$= \dfrac{1}{\tan 77°} = \dfrac{\sin 13°}{\sin 77°} = \dfrac{\cos 77°}{\cos 13°} = \dfrac{\frac{1}{\csc 13°}}{\frac{1}{\sec 13°}} = \dfrac{\sec 13°}{\csc 13°}$

5.3 Trigonometric Functions of Any Angle

An angle in standard position has its vertex at the origin and initial side on the positive x-axis, and is measured counter-clockwise.

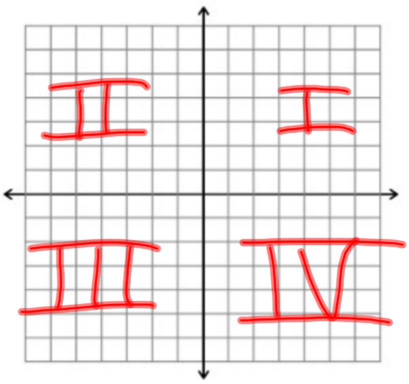


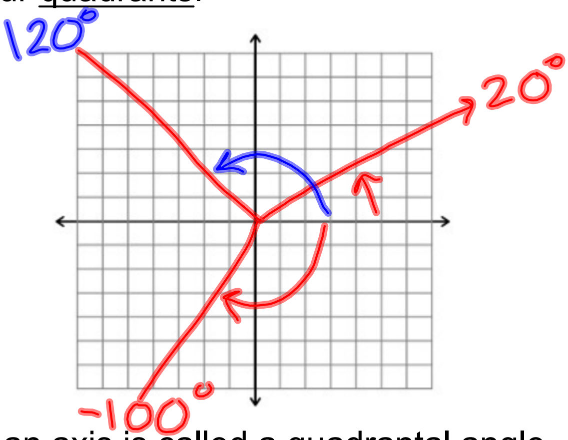
positive angle



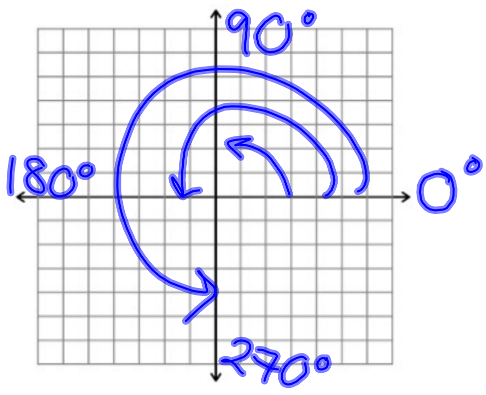
negative angle

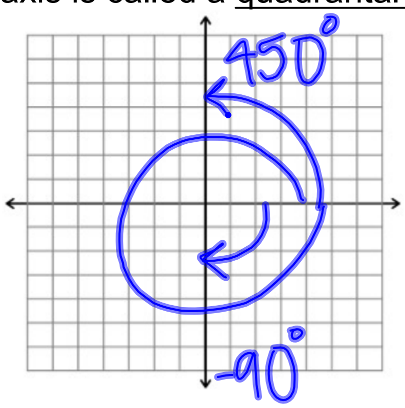
The coordinate plane is divided into four quadrants.



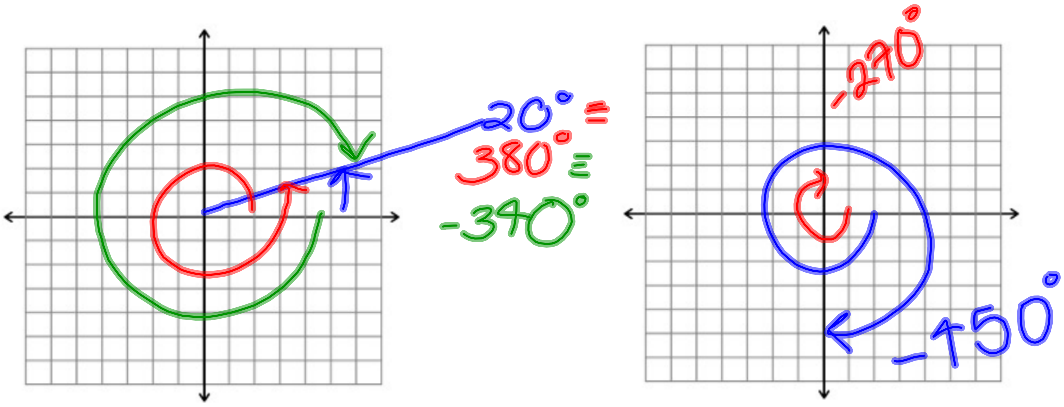


An angle whose terminal side falls on an axis is called a quadrantal angle.





Two angles sharing a terminal side are called coterminal and differ by integer multiples of 360°.

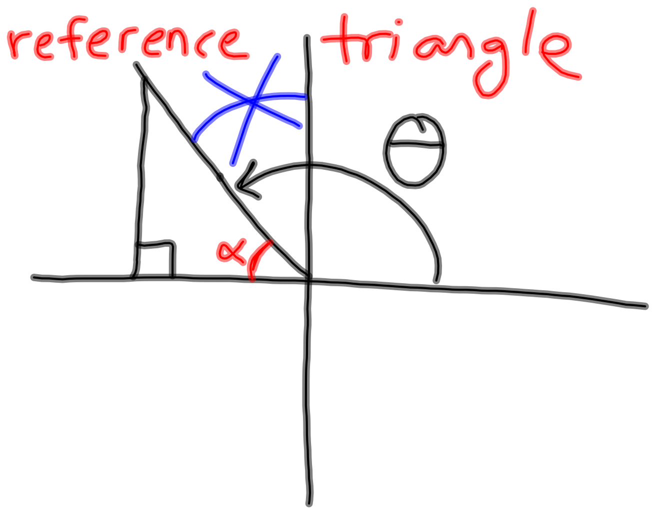


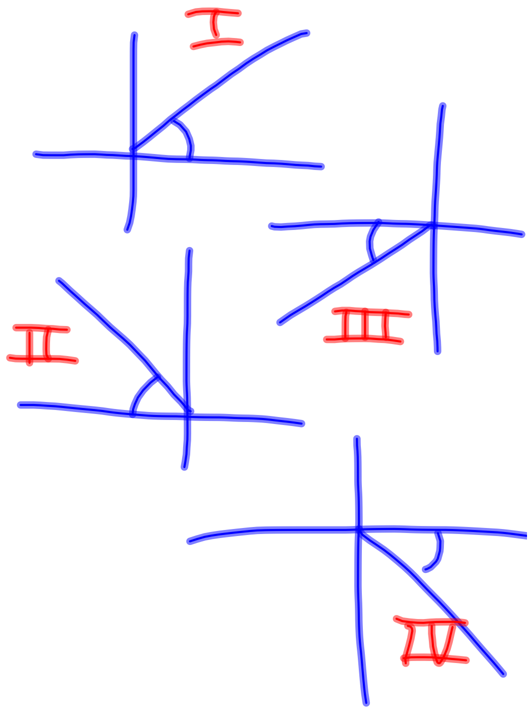
Find two positive and two negative angles that are coterminal with 89°.

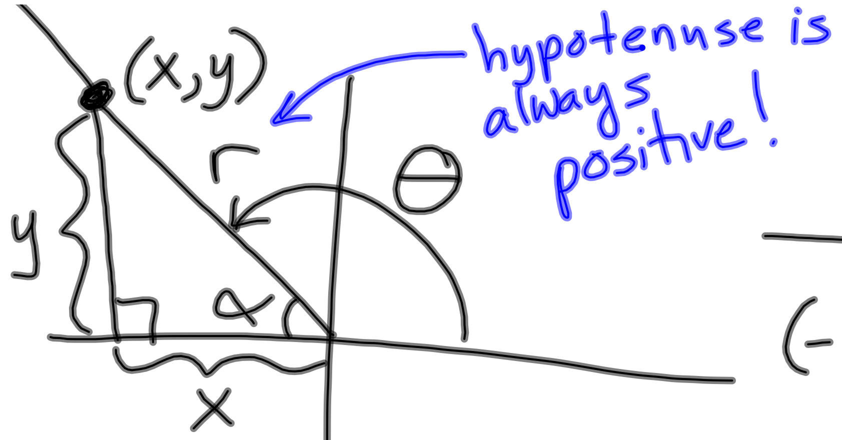
$89° + 360° =$

$89° - 360° =$

For an angle in standard position, the reference angle is the acute angle between the terminal side of the angle and the x-axis.



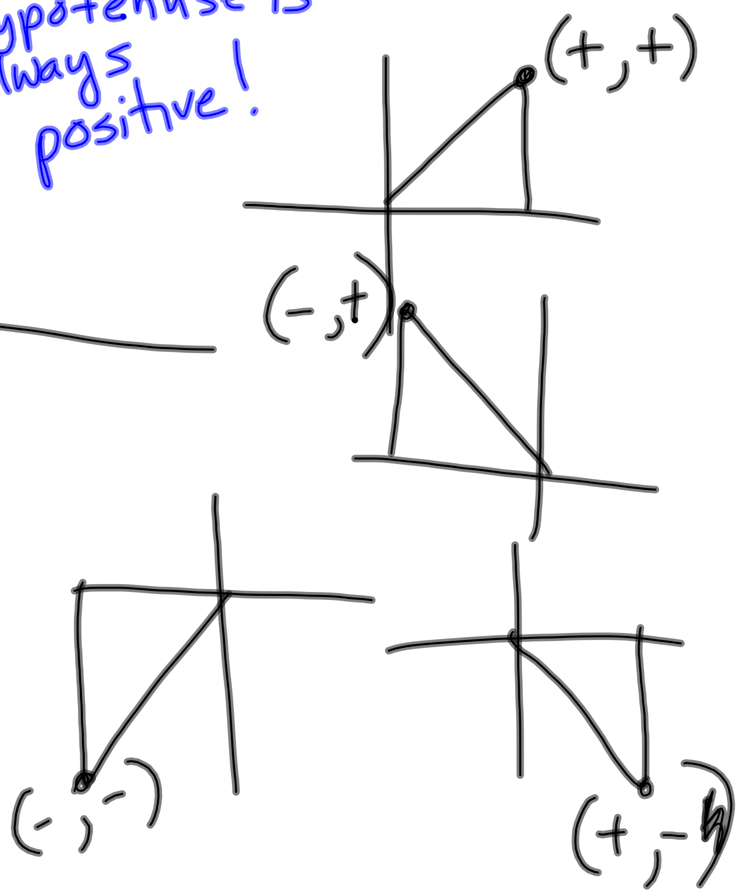




$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$

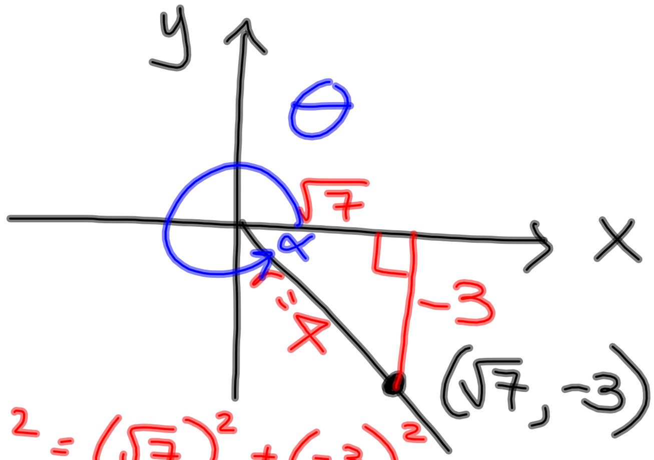
$\tan\theta = \frac{y}{x}$



II **Students** only sine (and its reciprocal)	**I** **All** all functions are positive
Take only tan (and its reciprocal) **III**	**Calculus** only cosine (and its reciprocal) **IV**

Tells us which functions are positive in which quadrants.

5.3 #26



$$\csc\theta = -\frac{4}{3}$$

$$\tan\theta = -\frac{3}{\sqrt{7}}$$

$$\cos\theta = \frac{\sqrt{7}}{4}$$

$$r^2 = (\sqrt{7})^2 + (-3)^2$$

$$r = \sqrt{7+9} = \sqrt{16} = 4$$

Homework:

5.1 #83-97 odd