

Find the exact value of the following.

$30^\circ < 60^\circ < 90^\circ$
 $1 < \sqrt{3} < 2$
 $\sqrt{1} < \sqrt{3} < \sqrt{4}$
 $1 < \sqrt{3} < 2$

a. $\sin 270^\circ = -1$
 $\cos \frac{2\pi}{3} = \frac{1}{2}$
 $\sin \frac{2\pi}{3} = \frac{\sqrt{3}}{2}$

b. $\cos -225^\circ = -\frac{1}{\sqrt{2}}$
 $\tan \frac{7\pi}{4} = -1$

c. $\sec 315^\circ = \sqrt{2}$
 $\sin \frac{7\pi}{6} = -\frac{1}{2}$
 $\cos \frac{7\pi}{6} = -\frac{\sqrt{3}}{2}$

d. $\csc 420^\circ = \frac{2}{\sqrt{3}}$
 $\csc(-180^\circ) = \frac{1}{\sin(-180^\circ)} = \frac{1}{0} = \text{undefined}$

e. $\cot -135^\circ = 1$

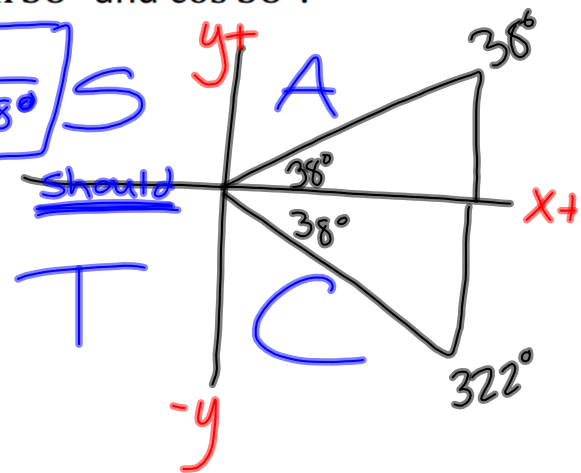
Unit Circle: $(\cos \theta, \sin \theta) = (x, y)$
 $\cos \theta = \frac{x}{r}$
 $\sin \theta = \frac{y}{r}$

7. Write the following in terms of $\sin 38^\circ$ and $\cos 38^\circ$.

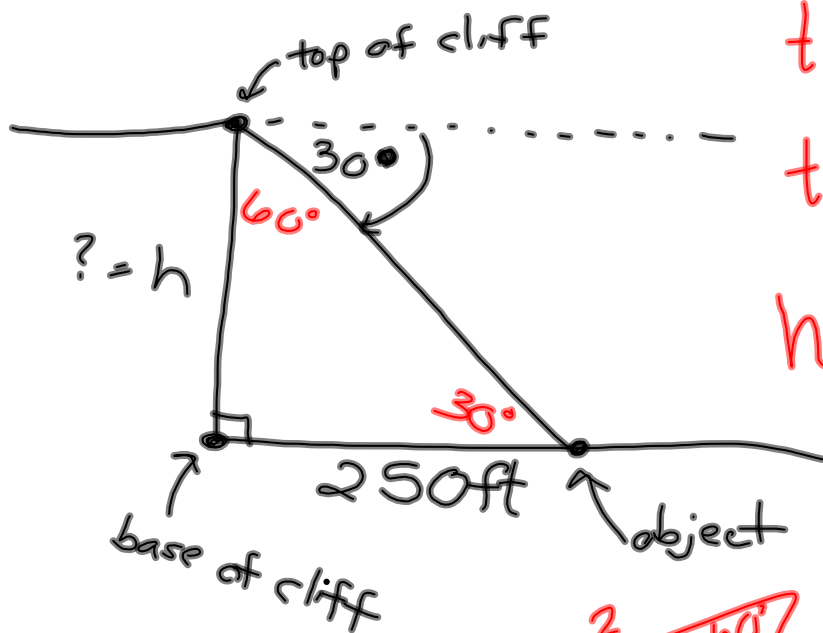
a. $\csc 322^\circ = -\csc 38^\circ = \frac{-1}{\sin 38^\circ}$

b. $\tan 52^\circ =$

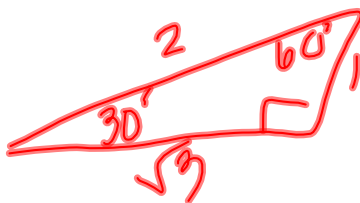
$\cot(90^\circ - 52^\circ) =$
 $= \cot 38^\circ =$
 $= \frac{\cos 38^\circ}{\sin 38^\circ}$



8. The angle of depression from the top of a cliff to an object on the ground is 30° . If the object is 250 feet from the base of the cliff, how tall is the cliff? Give an exact answer in feet.



$$\begin{aligned} \tan 60^\circ &= \frac{250}{h} \\ \tan 30^\circ &= \frac{h}{250} \\ h &= 250 \tan 30^\circ \\ &= 250 \cdot \frac{1}{\sqrt{3}} \\ &= \frac{250}{\sqrt{3}} \text{ ft} \end{aligned}$$



9. A child rides his tricycle at a rate of 20 miles per hour. If the diameter of the front wheel is 8 inches, find the angular speed of the wheel in revolutions per minute. Give an exact answer, in terms of π if necessary.

$$V = \frac{20 \text{ mi}}{h}; \quad r = 4 \text{ in}; \quad \omega = ? \frac{\text{rev}}{\text{min}}$$

$$\frac{V}{r} = \frac{r\omega}{r} \quad \omega = \frac{V}{r} = \frac{V}{1} \cdot \frac{1}{r} = \frac{20 \text{ mi}}{h} \cdot \frac{1}{4 \text{ in}}$$

$$\omega = \frac{20 \text{ mi}}{h} \cdot \frac{1}{4 \text{ in}} \cdot \frac{5280 \text{ ft}}{1 \text{ mi}} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{1 h}{60 \text{ min}} \cdot \frac{1 \text{ rev}}{2\pi}$$

$$= \frac{2640}{\pi} \text{ rev/min}$$

10. Find the exact measure in inches of the radius of a circle with a central angle of 72° that subtends an arc of length 8 feet.

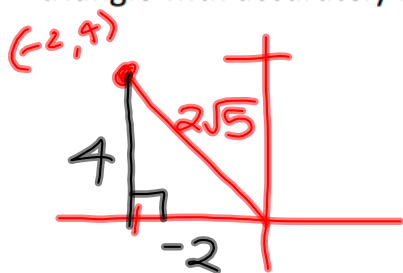
$$r = ? \text{ in} ; \theta = 72^\circ ; s = 8 \text{ ft}$$

$$\frac{s}{\theta} = \frac{r\theta}{\theta}$$

$$r = \frac{8 \text{ ft}}{72^\circ} \cdot \frac{12 \text{ in}}{1 \text{ ft}} \cdot \frac{60}{\pi} = \boxed{\frac{240}{\pi} \text{ in}}$$

5. Given that the terminal side of an angle β passes through the point $(-2, 4)$,

a. Draw a picture depicting the reference triangle with accurately labeled sides.



b. Find the length of the hypotenuse.
(simplify all radicals)

$$\begin{aligned} h^2 &= (4)^2 + (-2)^2 \\ &= 16 + 4 = 20 \\ h &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

c. Evaluate $\cot \beta$.

$$\frac{-2}{4} = \boxed{-\frac{1}{2}}$$

d. Evaluate $\csc \beta$.

$$\frac{2\sqrt{5}}{4} = \boxed{\frac{\sqrt{5}}{2}}$$

e. Evaluate $\cos \beta$.

$$\frac{-2}{2\sqrt{5}} = \boxed{-\frac{1}{\sqrt{5}}}$$

Formulas to know for Test #1:

Trig Functions of an Acute Angle

$$\sin \theta = \frac{\text{side opposite } \theta}{\text{hypotenuse}} \quad \csc \theta = \frac{\text{hypotenuse}}{\text{side opposite } \theta}$$

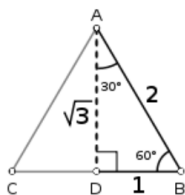
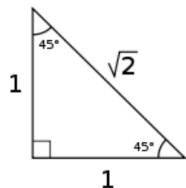
$$\cos \theta = \frac{\text{side adjacent to } \theta}{\text{hypotenuse}} \quad \sec \theta = \frac{\text{hypotenuse}}{\text{side adjacent to } \theta}$$

$$\tan \theta = \frac{\text{side opposite } \theta}{\text{side adjacent to } \theta} \quad \cot \theta = \frac{\text{side adjacent to } \theta}{\text{side opposite } \theta}$$

Converting Between Degree & Radian Measure

To convert from degree to radian measure, multiply by $\frac{\pi}{180^\circ}$

To convert from radian to degree measure, multiply by $\frac{180^\circ}{\pi}$



Trig Identities

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}, \quad \sin x = \frac{1}{\csc x}, \quad \sec x = \frac{1}{\cos x}, \quad \cos x = \frac{1}{\sec x}, \quad \cot x = \frac{1}{\tan x}, \quad \tan x = \frac{1}{\cot x}$$

Ratio Identities

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

Arc Length and Angular Speed

Variables

s = distance traveled or arc length (inches, kilometers, etc)
 t = time (seconds, minutes, hours, days, etc)
 θ = amount of rotation or included angle (degrees, radians, rotations, revolutions)
 r = radius or distance from the center of rotation (centimeters, inches, etc)
 v = linear speed = $\frac{\text{distance}}{\text{time}}$
 ω = angular speed = $\frac{\text{amount of rotation}}{\text{time}}$

Formulas

$$s = r\theta, \quad v = \frac{s}{t}, \quad \omega = \frac{\theta}{t}, \quad v = r\omega$$

Dimensional analysis conversion factors

$$\frac{5280 \text{ ft}}{1 \text{ mi}}, \quad \frac{12 \text{ in}}{1 \text{ ft}}, \quad \frac{2\pi}{1 \text{ rev}}, \quad \frac{\pi}{180^\circ}, \quad \frac{60 \text{ min}}{1 \text{ hr}}, \quad \frac{60 \text{ sec}}{1 \text{ min}}, \text{ and their reciprocals}$$

Key words:

- cofunction
- complement
- coterminal
- reference angle
- heading
- bearing
- angle of elevation/depression