

1. $\cos 3\pi = -1$

2. $\sec \frac{5\pi}{4} = -\sqrt{2}$

3. $\sin(-90^\circ) = -1$

4. $\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$

5. $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

6. $y = 2 \sin(\frac{1}{3}x) + 4$

amplitude: $\frac{2}{1}$

period: $\frac{6\pi}{1/3}$

horizontal shift: none

vertical shift: up 4

7. $y = -2 \cos(2x - \frac{\pi}{2})$

amplitude: $\frac{2}{1}$

period: $\frac{\pi}{2}$

horizontal shift: right $\frac{\pi}{4}$

vertical shift: none

8. $y = -\frac{2}{3} \sec(x - \pi)$

"amplitude": $\frac{2/3}{1}$

period: $\frac{2\pi}{1}$

horizontal shift: right π

vertical shift: none

9. $y = \csc 3x + \frac{1}{2}$

"amplitude": $\frac{1}{2\pi/3}$

period: $\frac{2\pi}{3}$

horizontal shift: none

vertical shift: up 1/2

12. $y = 2 \cos x + \sin x$

amp 2, amp 1

13. $y = \cos 2x - x$

per pi

10. $y = 2 \cot(x + \frac{\pi}{2})$

"amplitude": $\frac{2}{1}$

period: $\frac{\pi}{1}$

horizontal shift: left $\frac{\pi}{2}$

vertical shift: none

11. $y = -\frac{1}{2} \tan \frac{\pi}{2} x - 1$

"amplitude": $\frac{1/2}{1}$

period: $\frac{\pi}{2}$

horizontal shift: none

vertical shift: down 1

Function	Range	Different function with the same graph
(6.) $y = 2 \sin(\frac{1}{3}x) + 4$	$[2, 6]$	$y = 2 \cos(\frac{1}{3}x - \frac{\pi}{2}) + 4$
(7.) $y = -2 \cos(2x - \frac{\pi}{2})$	$[-2, 2]$	$y = -2 \sin 2x$
(8.) $y = -\frac{2}{3} \sec(x - \pi)$	$(-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, \infty)$	$y = \frac{2}{3} \sec x$
(9.) $y = \csc 3x + \frac{1}{2}$	$(-\infty, -\frac{1}{2}] \cup [\frac{3}{2}, \infty)$	$y = \sec(3x - \frac{\pi}{2}) + \frac{1}{2}$
(10.) $y = 2 \cot(x + \frac{\pi}{2})$	$(-\infty, \infty)$	$y = -2 \tan x$
(11.) $y = -\frac{1}{2} \tan \frac{\pi}{2} x - 1$	$(-\infty, \infty)$	$y = \frac{1}{2} \cot(\frac{\pi}{2} x \pm \frac{\pi}{2}) - 1$

6.1 Identities: Pythagorean & Sum and Difference

Reciprocal Identities

$$\csc x = \frac{1}{\sin x}, \quad \sin x = \frac{1}{\csc x}, \quad \sec x = \frac{1}{\cos x}, \quad \cos x = \frac{1}{\sec x}, \quad \cot x = \frac{1}{\tan x}, \quad \tan x = \frac{1}{\cot x}$$

Ratio Identities

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x, \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x, \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x, \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1, \quad 1 + \cot^2 x = \csc^2 x, \quad \tan^2 x + 1 = \sec^2 x$$

Sum and Difference Identities (6.1-book, 6.2-handout)

$$\sin(a+b) \neq \sin a + \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

6.2 handout

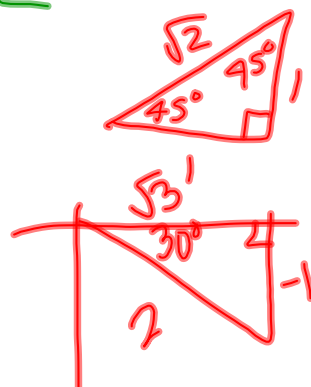
$$2. \sin 375^\circ = \sin (330^\circ + 45^\circ)$$

$$= \sin 330^\circ \cos 45^\circ + \cos 330^\circ \sin 45^\circ$$

$$= \left(-\frac{1}{2} \right) \left(\frac{\sqrt{2}}{2} \right) + \left(\frac{\sqrt{3}}{2} \right) \left(\frac{\sqrt{2}}{2} \right)$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$



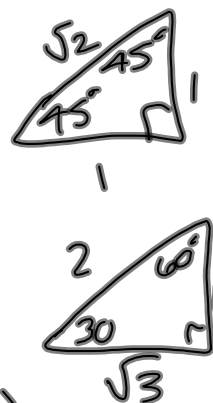
$$10. \cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) =$$

$$= \cos\frac{\pi}{4} \cos\frac{\pi}{3} + \sin\frac{\pi}{4} \sin\frac{\pi}{3}$$

$$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}$$



$$14. \sin^{a} 167^{\circ} \cos^{b} 107^{\circ} - \cos^{a} 167^{\circ} \sin^{b} 107^{\circ}$$

$$= \sin(167^{\circ} - 107^{\circ})$$

$$= \sin 60^{\circ}$$

$$= \boxed{\frac{\sqrt{3}}{2}}$$

$$20. \sin^a x \cos^b 3x + \cos^a x \sin^b 3x$$

$$= \sin(x+3x)$$

$$= \boxed{\sin 4x}$$

(34.) Given $\sin \alpha = \frac{24}{25}$, $\alpha \in \text{Q II}$

$$\cos \beta = \frac{-4}{5}, \beta \in \text{Q III}$$

Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.



$$\begin{aligned} \sin(\alpha-\beta) &= \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ &= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right) \\ &= \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}} \end{aligned}$$

$$\begin{aligned} \cos(\alpha-\beta) &= \cos \alpha \cos \beta + \sin \alpha \sin \beta \\ &= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right) \\ &= \frac{28}{125} - \frac{72}{125} = \boxed{\frac{-44}{125}} \end{aligned}$$

$$\begin{aligned} \tan(\alpha-\beta) &= \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{-\frac{117}{125}}{-\frac{44}{125}} = \frac{-117}{125} \cdot \frac{125}{-44} \\ &= \boxed{\frac{117}{44}} \end{aligned}$$

$$\alpha-\beta \in \boxed{\text{Q III}}$$

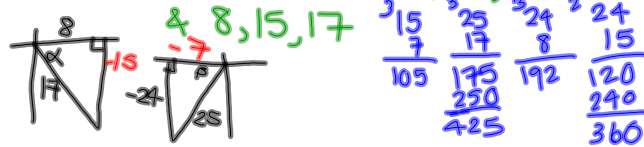
$$40. \text{ Given } \cos \alpha = \frac{8}{17}, \alpha \in \text{QIV}$$

$$\sin \beta = \frac{-24}{25}, \beta \in \text{QIII}$$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.

*Pythagorean triples that are useful to know:

3, 4, 5 ; 5, 12, 13 ; 7, 24, 25 ;



$$\sin(\alpha+\beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{24}{25}\right) = \frac{105}{425} - \frac{192}{425} = \frac{-87}{425}$$

$$\cos(\alpha+\beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \frac{-56}{425} - \frac{360}{425} = \frac{-416}{425}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{87}{416}$$

$$\alpha+\beta \in \text{Q III}$$

Homework:

6.1 Handout: #13-23 odd (proofs)

6.2 Handout: #1-23 odd; 35-41 odd

& **memorize your identities!!!**

6.1 Prove.

$$21. \frac{\cos x}{1-\sin x} = \sec x + \tan x$$

$$\text{LHS} = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x} = \dots$$