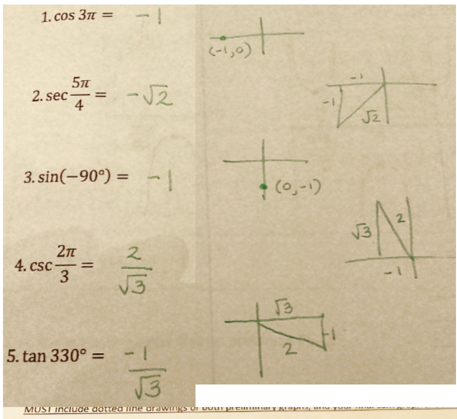
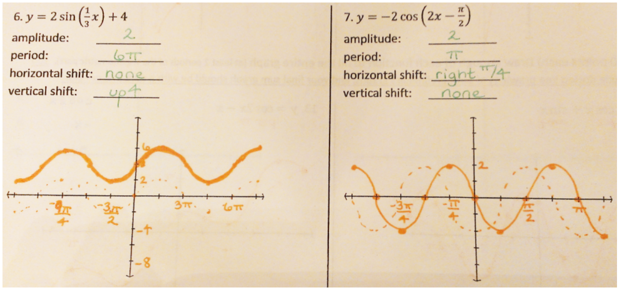
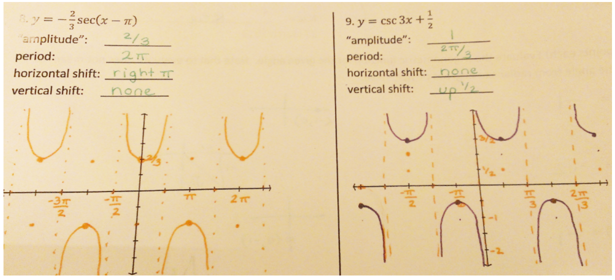
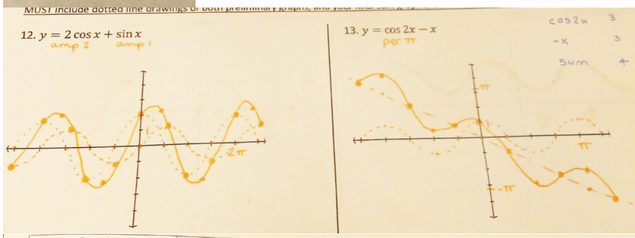
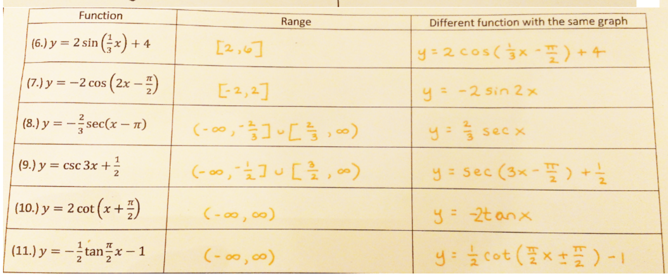


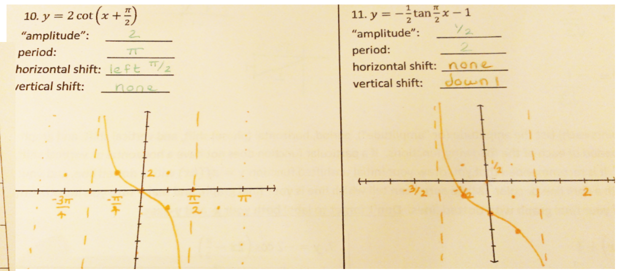








Function	Range	Different function with the same graph
(6.) $y = 2\sin\left(\frac{1}{3}x\right) + 4$	$[2, 6]$	$y = 2\cos\left(\frac{1}{3}x - \frac{\pi}{2}\right) + 4$
(7.) $y = -2\cos\left(2x - \frac{\pi}{2}\right)$	$[-2, 2]$	$y = -2\sin 2x$
(8.) $y = -\frac{2}{3}\sec(x - \pi)$	$(-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, \infty)$	$y = \frac{2}{3}\sec x$
(9.) $y = \csc 3x + \frac{1}{2}$	$(-\infty, -\frac{1}{2}] \cup [\frac{3}{2}, \infty)$	$y = \sec\left(3x - \frac{\pi}{2}\right) + \frac{1}{2}$
(10.) $y = 2\cot\left(x + \frac{\pi}{2}\right)$	$(-\infty, \infty)$	$y = -2\tan x$
(11.) $y = -\frac{1}{2}\tan\frac{\pi}{2}x - 1$	$(-\infty, \infty)$	$y = \frac{1}{2}\cot\left(\frac{\pi}{2}x \pm \frac{\pi}{2}\right) - 1$



6.1 Identities: Pythagorean & Sum and Difference

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \;,\; \sin x = \frac{1}{\csc x} \;,\; \sec x = \frac{1}{\cos x} \;,\; \cos x = \frac{1}{\sec x} \;,\; \cot x = \frac{1}{\tan x} \;,\; \tan x = \frac{1}{\cot x}$$

Ratio Identities

$$\tan x = \frac{\sin x}{\cos x} \;,\; \cot x = \frac{\cos x}{\sin x}$$

Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \quad , \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x \quad , \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x \quad , \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1 \quad , \quad 1 + \cot^2 x = \csc^2 x \quad , \quad \tan^2 x + 1 = \sec^2 x$$

<u>Sum and Difference Identities</u> (6.1-book, 6.2-handout)

$\sin(a+b) \neq \sin a + \sin b$

$\sin(a+b) = \sin a \cos b + \cos a \sin b$

$\sin(a-b) = \sin a \cos b - \cos a \sin b$

$\cos(a+b) = \cos a \cos b - \sin a \sin b$

$\cos(a-b) = \cos a \cos b + \sin a \sin b$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

<u>6.2 handout</u>

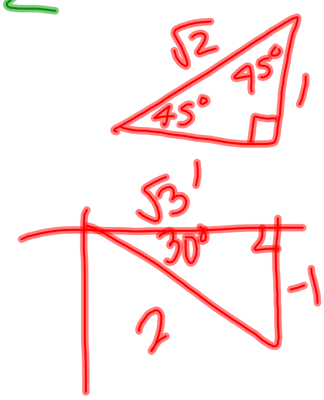
2. $\sin 375^\circ = \sin(\overset{a}{330^\circ} + \overset{b}{45^\circ})$

$= \sin 330^\circ \cos 45^\circ + \cos 330^\circ \sin 45^\circ$

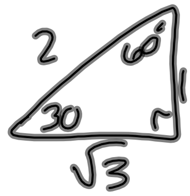
$= \left(\frac{-1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{6} - \sqrt{2}}{4}}$



10. $\cos\left(\frac{\pi}{4} - \frac{\pi}{3}\right) =$



$= \cos\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{4}\sin\frac{\pi}{3}$

$= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$

14. $\sin\overset{a}{167^\circ}\cos\overset{b}{107^\circ} - \cos\overset{a}{167^\circ}\sin\overset{b}{107^\circ}$

$= \sin(167^\circ - 107^\circ)$

$= \sin 60^\circ$

$= \boxed{\frac{\sqrt{3}}{2}}$

20. $\sin \overset{a}{x} \cos \overset{b}{3x} + \cos \overset{a}{x} \sin \overset{b}{3x}$

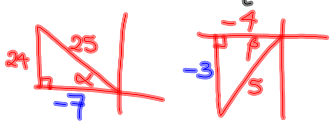
$= \sin(x + 3x)$

$= \boxed{\sin 4x}$

(34.) Given $\sin\alpha = \frac{24}{25}$, $\alpha \in QII$

$\cos\beta = \frac{-4}{5}$, $\beta \in QIII$

Find $\sin(\alpha-\beta)$, $\cos(\alpha-\beta)$, $\tan(\alpha-\beta)$ & determine the quadrant in which $\alpha-\beta$ lies.



$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(\frac{24}{25}\right)\left(\frac{-4}{5}\right) - \left(\frac{-7}{25}\right)\left(\frac{-3}{5}\right)$

$= \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}}$

$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$= \left(\frac{-7}{25}\right)\left(\frac{-4}{5}\right) + \left(\frac{24}{25}\right)\left(\frac{-3}{5}\right)$

$= \frac{28}{125} - \frac{72}{125} = \boxed{\frac{-44}{125}}$

$\tan(\alpha-\beta) = \frac{\sin(\alpha-\beta)}{\cos(\alpha-\beta)} = \frac{\frac{-117}{125}}{\frac{-44}{125}} = \frac{-117}{\cancel{125}} \cdot \frac{\cancel{125}}{-44}$

$= \boxed{\frac{117}{44}}$

$\alpha-\beta \in$ QIII

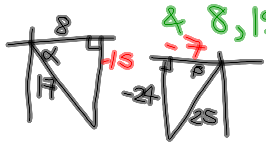
40. Given $\cos\alpha = \frac{8}{17}$, $\alpha \in Q\,IV$

$\sin\beta = \frac{-24}{25}$, $\beta \in Q\,III$

find $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$, $\tan(\alpha+\beta)$, & determine the quadrant in which $\alpha+\beta$ lies.

*Pythagorean triples that are useful to know:

3,4,5 ; 5,12,13 ; 7,24,25 ; & 8,15,17



$$\begin{array}{r} 15 \\ \times\ 7 \\ \hline 105 \end{array} \qquad \begin{array}{r} 25 \\ \times\ 17 \\ \hline 175 \\ 250 \\ \hline 425 \end{array} \qquad \begin{array}{r} 24 \\ \times\ 8 \\ \hline 192 \end{array} \qquad \begin{array}{r} 24 \\ \times\ 15 \\ \hline 120 \\ 240 \\ \hline 360 \end{array}$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$
$$= \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right) = \frac{105}{425} - \frac{192}{425} = \boxed{\frac{-87}{425}}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$
$$= \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \frac{-56}{425} - \frac{360}{425} = \boxed{\frac{-416}{425}}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \boxed{\frac{87}{416}}$$

$$\alpha+\beta \in \boxed{Q\,III}$$

Homework:

6.1 Handout: #13-23 odd (proofs)

6.2 Handout: #1-23 odd; 35-41 odd

& **memorize your identities!!!**

6.1 Prove.

21. $\frac{\cos x}{1-\sin x} = \sec x + \tan x$

$$LHS = \frac{\cos x}{1-\sin x} \cdot \frac{1+\sin x}{1+\sin x} = \frac{\cos x(1+\sin x)}{1-\sin^2 x} = \ldots$$