

1.  $\cos 3\pi = -1$

2.  $\sec \frac{5\pi}{4} = -\sqrt{2}$

3.  $\sin(-90^\circ) = -1$

4.  $\csc \frac{2\pi}{3} = \frac{2}{\sqrt{3}}$

5.  $\tan 330^\circ = -\frac{1}{\sqrt{3}}$

6.  $y = 2 \sin(\frac{1}{3}x) + 4$

amplitude:  $\frac{2}{1}$

period:  $\frac{6\pi}{1/3}$

horizontal shift: none

vertical shift: up 4

7.  $y = -2 \cos(2x - \frac{\pi}{2})$

amplitude:  $\frac{2}{1}$

period:  $\frac{\pi}{2}$

horizontal shift: right  $\frac{\pi}{4}$

vertical shift: none

12.  $y = 2 \cos x + \sin x$

amp 2, amp 1

13.  $y = \cos 2x - x$

cos 2x, -x

8.  $y = -\frac{2}{3} \sec(x - \pi)$

amplitude:  $\frac{2/3}{1}$

period:  $\frac{2\pi}{1}$

horizontal shift: right  $\pi$

vertical shift: none

9.  $y = \csc 3x + \frac{1}{2}$

amplitude:  $\frac{1}{2\pi/3}$

period:  $\frac{2\pi}{3}$

horizontal shift: none

vertical shift: up  $\frac{1}{2}$

10.  $y = 2 \cot(x + \frac{\pi}{2})$

amplitude:  $\frac{2}{1}$

period:  $\frac{\pi}{1}$

horizontal shift: left  $\frac{\pi}{2}$

vertical shift: none

11.  $y = -\frac{1}{2} \tan \frac{\pi}{2} x - 1$

amplitude:  $\frac{1/2}{1}$

period:  $\frac{\pi}{1/2}$

horizontal shift: none

vertical shift: down 1

Function	Range	Different function with the same graph
(6.) $y = 2 \sin(\frac{1}{3}x) + 4$	$[2, 6]$	$y = 2 \cos(\frac{1}{3}x - \frac{\pi}{2}) + 4$
(7.) $y = -2 \cos(2x - \frac{\pi}{2})$	$[-2, 2]$	$y = -2 \sin 2x$
(8.) $y = -\frac{2}{3} \sec(x - \pi)$	$(-\infty, -\frac{2}{3}] \cup [\frac{2}{3}, \infty)$	$y = \frac{2}{3} \sec x$
(9.) $y = \csc 3x + \frac{1}{2}$	$(-\infty, -\frac{1}{2}] \cup [\frac{3}{2}, \infty)$	$y = \sec(3x - \frac{\pi}{2}) + \frac{1}{2}$
(10.) $y = 2 \cot(x + \frac{\pi}{2})$	$(-\infty, \infty)$	$y = -2 \tan x$
(11.) $y = -\frac{1}{2} \tan \frac{\pi}{2} x - 1$	$(-\infty, \infty)$	$y = \frac{1}{2} \cot(\frac{\pi}{2} x \pm \frac{\pi}{2}) - 1$

## 6.1 Identities: Pythagorean & Sum and Difference

### Reciprocal Identities

$$\csc x = \frac{1}{\sin x}, \quad \sin x = \frac{1}{\csc x}, \quad \sec x = \frac{1}{\cos x}, \quad \cos x = \frac{1}{\sec x}, \quad \cot x = \frac{1}{\tan x}, \quad \tan x = \frac{1}{\cot x}$$

### Ratio Identities

$$\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}$$

### Cofunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x, \quad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x, \quad \cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x, \quad \sec\left(\frac{\pi}{2} - x\right) = \csc x$$

### Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1, \quad 1 + \cot^2 x = \csc^2 x, \quad \tan^2 x + 1 = \sec^2 x$$

Sum and Difference Identities (6.1-book, 6.2-handout)

$$\sin(a+b) \neq \sin a + \sin b$$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$

$$\sin(a-b) = \sin a \cos b - \cos a \sin b$$

$$\cos(a+b) = \cos a \cos b - \sin a \sin b$$

$$\cos(a-b) = \cos a \cos b + \sin a \sin b$$

$$\tan(a+b) = \frac{\tan a + \tan b}{1 - \tan a \tan b}$$

$$\tan(a-b) = \frac{\tan a - \tan b}{1 + \tan a \tan b}$$

6.2 handout

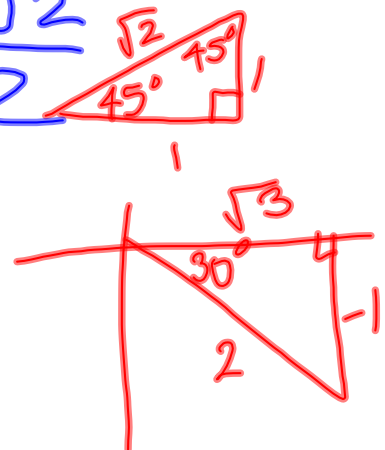
$$2. \sin 375^\circ = \sin(330^\circ + 45^\circ)$$

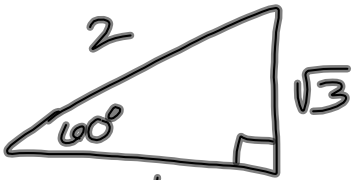
$$= \sin 330^\circ \cos 45^\circ + \cos 330^\circ \sin 45^\circ$$

$$= \frac{-1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$$

$$= \frac{-\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}$$





$$\begin{aligned}
 10. \quad \cos\left(\frac{\overset{a}{\pi}}{4} - \frac{\overset{b}{\pi}}{3}\right) &= \\
 &= \cos\frac{\pi}{4} \cos\frac{\pi}{3} + \sin\frac{\pi}{4} \sin\frac{\pi}{3} \\
 &= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} \\
 &= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} = \boxed{\frac{\sqrt{2} + \sqrt{6}}{4}}
 \end{aligned}$$


$$\begin{aligned}
 14. \quad \sin\overset{a}{167}^\circ \cos\overset{b}{107}^\circ - \cos\overset{a}{167}^\circ \sin\overset{b}{107}^\circ \\
 &= \sin(167^\circ - 107^\circ) \\
 &= \sin 60^\circ \\
 &= \boxed{\frac{\sqrt{3}}{2}}
 \end{aligned}$$

$$20. \sin x \cos 3x + \cos x \sin 3x$$

$$= \sin(x + 3x)$$

$$= \boxed{\sin 4x}$$

(34) Given  $\sin \alpha = \frac{24}{25}$ ,  $\alpha \in \text{Q II}$    $\cos \beta = \frac{-7}{5}$ ,  $\beta \in \text{Q III}$  

Find  $\sin(\alpha - \beta)$ ,  $\cos(\alpha - \beta)$ ,  $\tan(\alpha - \beta)$  &

① determine the quadrant in which  $\alpha - \beta$  lies.

$$\sqrt{25^2 - 24^2} = \sqrt{625 - 576} = \sqrt{49} = 7$$

$$\begin{array}{r} 124 \\ 24 \\ \hline 96 \\ 780 \\ \hline 576 \end{array} \quad \begin{array}{l} \textcircled{1} \sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta \\ = \frac{24}{25} \cdot \left(\frac{-7}{5}\right) - \left(\frac{-7}{25}\right) \left(\frac{-3}{5}\right) \\ = \frac{-96}{125} - \frac{21}{125} = \boxed{\frac{-117}{125}} \end{array}$$

$$\textcircled{2} \cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$= \left(\frac{-7}{25}\right) \left(\frac{-7}{5}\right) + \left(\frac{24}{25}\right) \left(\frac{-3}{5}\right)$$

$$= \frac{28}{125} - \frac{72}{125}$$

$$= \boxed{\frac{-44}{125}}$$

$$\textcircled{3} \tan(\alpha - \beta) = \frac{\sin(\alpha - \beta)}{\cos(\alpha - \beta)} = \frac{\frac{-117}{125}}{\frac{-44}{125}}$$

$$= \frac{-117}{125} \cdot \frac{125}{-44} = \boxed{\frac{117}{44}}$$

④  $\alpha - \beta$  is in  $\boxed{\text{Q III}}$

$$40. \text{ Given } \cos \alpha = \frac{8}{17}, \alpha \in \text{QIV}$$

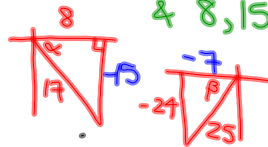
$$\sin \beta = \frac{-24}{25}, \beta \in \text{QIII}$$

find  $\sin(\alpha+\beta)$ ,  $\cos(\alpha+\beta)$ ,  $\tan(\alpha+\beta)$ , & determine the quadrant in which  $\alpha+\beta$  lies.

\*Pythagorean triples that are useful to know:

3,4,5 ; 5,12,13 ; 7,24,25 ;

& 8,15,17



$$\begin{array}{r} 3 \cdot 15 = 45 \\ 4 \cdot 24 = 96 \\ \hline 5 \cdot 25 = 125 \end{array} \quad \begin{array}{r} 3 \cdot 24 = 72 \\ 4 \cdot 15 = 60 \\ \hline 5 \cdot 25 = 125 \end{array} \quad \begin{array}{r} 24 \cdot 25 = 600 \\ 15 \cdot 17 = 255 \\ \hline 25 \cdot 25 = 625 \end{array}$$

$$\sin(\alpha+\beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta = \left(\frac{-15}{17}\right)\left(\frac{-7}{25}\right) + \left(\frac{8}{17}\right)\left(\frac{-24}{25}\right) = \frac{105 - 192}{425} = \frac{-87}{425}$$

$$\cos(\alpha+\beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta = \left(\frac{8}{17}\right)\left(\frac{-7}{25}\right) - \left(\frac{-15}{17}\right)\left(\frac{-24}{25}\right) = \frac{-56 - 360}{425} = \frac{-416}{425}$$

$$\tan(\alpha+\beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{87}{416}$$

$$\alpha+\beta \in \text{QIII}$$

### Homework:

6.1 Handout: #13-23 odd (proofs)

6.2 Handout: #1-23 odd; 35-41 odd

& **memorize your identities!!!**

6.1 Prove.

$$21. \frac{\cos x}{1 - \sin x} = \sec x + \tan x$$

$$\text{LHS} = \frac{\cos x}{1 - \sin x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{\cos x (1 + \sin x)}{1 - \sin^2 x} = \dots$$