Review: Which identities do you know?

$\frac{1}{\csc x} =$ $\sin x$

$\frac{\cos x}{\sin x} =$ $\cot x$

$\tan^2 x + 1 =$ $\sec^2 x$

$\sin(a - b) =$ $\sin a\cos b - \cos a\sin b$

$\cos a \cos b - \sin a \sin b =$ $\cos(a+b)$

$\frac{\tan a - \tan b}{1 + \tan a \tan b} =$ $\tan(a-b)$

Homework questions?

$(a-b)(a+b) = a^2 - b^2$

6.1

17. $(\sin x - \cos x)(\sin x + \cos x) = 1 - 2\cos^2 x$

LHS $= \sin^2 x - \cos^2 x$

$= 1 - \cos^2 x - \cos^2 x$

$= 1 - 2\cos^2 x$

$=$ RHS

$\sin^2 x + \cos^2 x = 1$

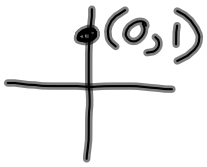
$\sin^2 x = 1 - \cos^2 x$

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90° - \theta$ or θ & $\frac{\pi}{2} - \theta$ are complementary angles

$$\cos\left(\frac{\pi}{2} - x\right) = \cos\frac{\pi}{2}\cos x + \sin\frac{\pi}{2}\sin x$$



$$= 0 \cdot \cos x + 1 \cdot \sin x$$

$$= \sin x$$

Double-Angle Identities

$$\sin(2\theta) = \sin(\theta + \theta)$$

$$= \sin\theta\cos\theta + \cos\theta\sin\theta$$

$$\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 6\theta = \sin 2(3\theta) = 2\sin 3\theta\cos 3\theta$

$\sin 8\theta = \sin 2(4\theta) = 2\sin 4\theta\cos 4\theta$

$\sin 44\theta = \sin 2(22\theta) = 2\sin 22\theta\cos 22\theta$

$\sin 3\theta = \sin 2\left(\frac{3\theta}{2}\right) = 2\sin\frac{3\theta}{2}\cos\frac{3\theta}{2}$

$= \sin(2\theta + \theta)$

even multiple ⇒ double $\angle$'s

odd multiple ⇒ sum

$\cos 2\theta = \cos(\theta+\theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1-\cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$= 1-\sin^2\theta - \sin^2\theta$

$\cos 2\theta = 1-2\sin^2\theta$

$\sin^2\theta+\cos^2\theta=1$

$\sin^2\theta = 1-\cos^2\theta$

$\cos^2\theta = 1-\sin^2\theta$

$\tan 2\theta = \tan(\theta+\theta)$

$= \dfrac{\tan\theta+\tan\theta}{1-\tan\theta\tan\theta}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Double-Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

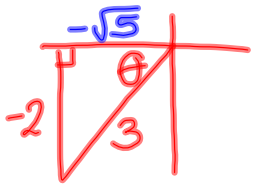
Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right)$

$\sin 2\theta = \frac{4\sqrt{5}}{9}$



$2\theta \in QI$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9} = \frac{1}{9} = \cos 2\theta$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = 4\sqrt{5} = \tan 2\theta$

Half-Angle Identities

$\sin\frac{x}{2} = ?$ | $\cos\frac{x}{2} = ?$

$\cos 2\theta = 1 - 2\sin^2\theta$

Let $\theta = \frac{x}{2}$.

$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$

$\cos x = 1 - 2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = 1 - \cos x$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

$\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = \frac{x}{2}$

$\cos 2\left(\frac{x}{2}\right) = 2\cos^2\frac{x}{2} - 1$

$\cos x + 1 = 2\cos^2\frac{x}{2}$

$\frac{\cos x + 1}{2} = \cos^2\frac{x}{2}$

$\pm\sqrt{\frac{1+\cos x}{2}} = \cos\frac{x}{2}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$$\tan\frac{7\pi}{12} = \tan\frac{\left(\frac{7\pi}{6}\right)}{2}$$

$$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$$

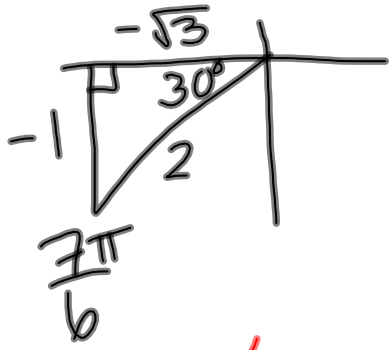
$$= \frac{1-\left(-\frac{\sqrt{3}}{2}\right)}{\frac{-1}{2}}$$

$$= \left(1+\frac{\sqrt{3}}{2}\right)\left(\frac{-2}{1}\right)$$

$$= -2-\sqrt{3}$$

$$\frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$



$\cos\frac{7\pi}{12}$ / $\sin\frac{7\pi}{12}$?

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Homework:

6.3 Handout #1-24; 30-36

& Memorize Identities!!!

(Quiz Friday)