Review: Which identities do you know?

$\frac{1}{\csc x} =$ sin x

$\frac{\cos x}{\sin x} =$ cot x

$\tan^2 x + 1 =$ $\sec^2 x$

$\sin(a - b) =$ sinacosb − cosasinb

$\cos a \cos b - \sin a \sin b =$ cos(a+b)

$\frac{\tan a - \tan b}{1 + \tan a \tan b} =$ tan(a−b)

6.2

5. $\tan(45° - 30°)$

$= \frac{\tan 45° - \tan 30°}{1 + \tan 45° \tan 30°}$

$= \frac{1 - \frac{\sqrt{3}}{3}}{1 + (1)(\frac{\sqrt{3}}{3})}$

Cofunction Identities

The function of an angle is equal to the cofunction of its complement.

θ & $90° - \theta$, θ & $\frac{\pi}{2} - \theta$ are complements

$\cos\left(\frac{\pi}{2} - \theta\right)$

$= \cos\frac{\pi}{2}\cos\theta + \sin\frac{\pi}{2}\sin\theta$

$= 0 \cdot \cos\theta + 1 \cdot \sin\theta$

$= \sin\theta$

Double-Angle Identities

$\sin(2\theta) = \sin(\theta + \theta)$

$= \sin\theta\cos\theta + \cos\theta\sin\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

The sine of twice any angle is equal to two times the sine of that angle times the cosine of that angle.

$\sin 4\theta = \sin 2(2\theta) = 2\sin 2\theta\cos 2\theta$

$\sin 6\theta = \sin 2(3\theta) = 2\sin 3\theta\cos 3\theta$

$\sin 52\theta = \sin 2(26\theta) = 2\sin 26\theta\cos 26\theta$

$\sin 3\theta = \sin 2\left(\frac{3\theta}{2}\right)$

$= 2\sin\frac{3\theta}{2}\cos\frac{3\theta}{2}$

$= \sin(2\theta + \theta)$

even multiples ⇒ double ∠

odd multiples ⇒ sum

$\cos 2\theta = \cos(\theta + \theta)$

$= \cos\theta\cos\theta - \sin\theta\sin\theta$

$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \cos^2\theta - (1 - \cos^2\theta)$

$\cos 2\theta = 2\cos^2\theta - 1$

$= (1 - \sin^2\theta) - \sin^2\theta$

$\cos 2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

$\tan 2\theta = \tan(\theta + \theta)$

$= \dfrac{\tan\theta + \tan\theta}{1 - \tan\theta\tan\theta}$

$\tan 2\theta = \dfrac{2\tan\theta}{1 - \tan^2\theta}$

Double-Angle Identities

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

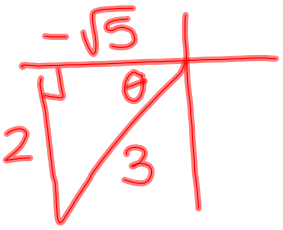
Given $\sin\theta = -\frac{2}{3}$, $\theta \in QIII$,

Find $\sin 2\theta$, $\cos 2\theta$, $\tan 2\theta$, and the quadrant in which 2θ lies.

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{-2}{3}\right)\left(\frac{-\sqrt{5}}{3}\right)$

$\sin 2\theta = \boxed{\frac{4\sqrt{5}}{9}}$



$\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(\frac{-\sqrt{5}}{3}\right)^2 - \left(\frac{-2}{3}\right)^2$

$= \frac{5}{9} - \frac{4}{9} = \boxed{\frac{1}{9}} = \cos 2\theta$

$\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta} = \boxed{4\sqrt{5}}$

$2\theta \in \boxed{QI}$

Half-Angle Identities

$\sin\frac{x}{2} = ?$ | $\cos\frac{x}{2} = ?$

$\cos 2\theta = 1-2\sin^2\theta$ & $\cos 2\theta = 2\cos^2\theta - 1$

Let $\theta = \frac{x}{2}$

$\cos 2\left(\frac{x}{2}\right) = 1-2\sin^2\left(\frac{x}{2}\right)$

$\cos x = 1-2\sin^2\frac{x}{2}$

$2\sin^2\frac{x}{2} = 1-\cos x$

$\sin^2\frac{x}{2} = \frac{1-\cos x}{2}$

$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$

Let $\theta = \frac{x}{2}$

$\cos 2\left(\frac{x}{2}\right)+1 = 2\cos^2\frac{x}{2}$

$\frac{\cos x+1}{2} = \cos^2\frac{x}{2}$

$\pm\sqrt{\frac{1+\cos x}{2}} = \cos\frac{x}{2}$

$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}} \quad , \quad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$= \frac{\sin x}{1+\cos x}$$

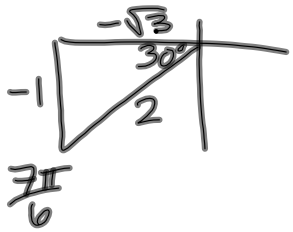
$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

$\tan\frac{7\pi}{12} = \tan\left(\frac{\frac{7\pi}{6}}{2}\right)$

$\frac{7\pi}{12} = \frac{1}{2}\left(\frac{7\pi}{6}\right)$

$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$

$= \frac{1-\cos\frac{7\pi}{6}}{\sin\frac{7\pi}{6}}$



$= \frac{1-\left(\frac{-\sqrt{3}}{2}\right)}{\frac{-1}{2}}$

$= \left(1+\frac{\sqrt{3}}{2}\right)\left(\frac{-2}{1}\right)$

$= -2-\sqrt{3}$

Homework:

6.3 Handout #1-24; 30-36

& Memorize Identities!!!

(Quiz Friday)