

Turn in homework:

6.1 Handout: #13-23 odd

6.2 Handout: #1-23 odd; 35-41 odd

6.3 Handout: #1-24; 30-36 all

(everything except 6.3 proofs)6.3 handout

$$\sin 2\theta = 2\sin\theta\cos\theta$$

66. $\sin 4x = 4\sin x \cos x - 8\cos x \sin^3 x$

$$\text{LHS} = \sin 2(2x) = 2 \sin 2x \cos 2x =$$

$$= 2 \underbrace{(2\sin x \cos x)}_{4\sin x \cos x} (1 - 2\sin^2 x)$$

$$= 4\sin x \cos x - 8\sin^3 x \cos x$$

$$= \text{RHS} \quad \square$$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 2\cos^2 x - 1 \\ &= 1 - 2\sin^2 x \end{aligned}$$

$$68. \sin 3x + \sin x = 4\sin x - 4\sin^3 x$$

$$\begin{aligned} \text{LHS} &= \sin(2x+x) + \sin x = \\ &= \underline{\sin 2x} \cos x + \underline{\cos 2x} \sin x + \sin x = \\ &= (2\sin x \cos x) \cos x + (\cos^2 x - \sin^2 x) \sin x + \sin x = \\ &= 2\sin x \cos^2 x + \sin x \cos^2 x - \sin^3 x + \sin x = \\ &= 3\sin x \cos^2 x - \sin^3 x + \sin x = \\ &= \overset{\substack{\sin^2 x + \cos^2 x = 1 \\ \cos^2 x = 1 - \sin^2 x}}{3\sin x (1 - \sin^2 x)} - \sin^3 x + \sin x = \\ &= 3\sin x - 3\sin^3 x - \sin^3 x + \sin x = \\ &= 4\sin x - 4\sin^3 x \\ &= \text{RHS.} \end{aligned}$$

$$72. \cos^2 \frac{x}{2} = \frac{\sec x + 1}{2 \sec x}$$

$$\begin{aligned} \text{LHS} &= \left(\cos \frac{x}{2} \right)^2 = \left(\pm \sqrt{\frac{1 + \cos x}{2}} \right)^2 = \\ &= \frac{1 + \cos x}{2} \overset{\substack{\sec x \\ \frac{1}{\sec x}}}{=} + \frac{1}{\sec x} = \\ &= \frac{\left(\frac{\sec x + 1}{\sec x} \right)}{2} = \frac{\sec x + 1}{\sec x} \cdot \frac{1}{2} = \frac{\sec x + 1}{2 \sec x} \\ &= \text{RHS.} \end{aligned}$$

$$76. \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = \cos x$$

$$\begin{aligned} \text{LHS} &= \left(\cos \frac{x}{2} \right)^2 - \left(\sin \frac{x}{2} \right)^2 = \\ &= \left(\pm \sqrt{\frac{1+\cos x}{2}} \right)^2 - \left(\pm \sqrt{\frac{1-\cos x}{2}} \right)^2 = \\ &= \frac{1+\cos x}{2} - \frac{1-\cos x}{2} = \\ &= \frac{1+\cos x - (1-\cos x)}{2} = \\ &= \frac{2\cos x}{2} = \cos x = \text{RHS} \quad \square \end{aligned}$$

$$86. \frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$$

$$\begin{aligned} 1 + \cot^2 x &= \csc^2 x \\ \cot^2 x &= \csc^2 x - 1 \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \frac{1 - 2\sin^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} - \frac{2\sin^2 x}{\sin^2 x} \\ &= \csc^2 x - 2 = \text{RHS} \quad \square \end{aligned}$$

$$\begin{aligned} \text{LHS} &= \frac{\cos^2 x - \sin^2 x}{\sin^2 x} = \frac{\cos^2 x}{\sin^2 x} - \frac{\sin^2 x}{\sin^2 x} = \cot^2 x - 1 = \\ &= \csc^2 x - 1 - 1 = \csc^2 x - 2 = \text{RHS} \quad \square \end{aligned}$$

$$88. \frac{2 \cos 2x}{\sin 2x} = \cot x - \tan x$$

$$\text{LHS} = \frac{2(\cos^2 x - \sin^2 x)}{2 \sin x \cos x} = \frac{\cos^2 x}{\sin x \cos x} - \frac{\sin^2 x}{\sin x \cos x}$$

$$= \frac{\cos x}{\sin x} - \frac{\sin x}{\cos x} = \cot x - \tan x = \text{RHS.}$$

Homework:

Continue working on 6.3 proofs