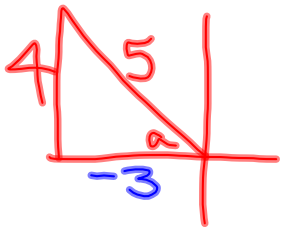
Review:

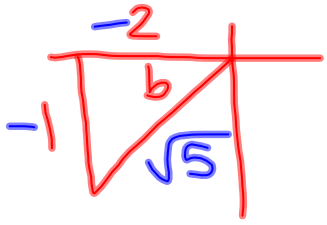
Given $\csc a = \frac{5}{4}, a \in QII$, and $\tan b = \frac{1}{2}, b \in QIII$, find:



$\cos(a-b) = \cos a\cos b + \sin a\sin b$

$= \left(\frac{-3}{5}\right)\left(\frac{-2}{\sqrt{5}}\right) + \left(\frac{4}{5}\right)\left(\frac{-1}{\sqrt{5}}\right)$

$= \frac{6}{5\sqrt{5}} - \frac{4}{5\sqrt{5}} = \boxed{\frac{2}{5\sqrt{5}}}$



$\sin(2a) = 2\sin a\cos a$

$= 2\left(\frac{4}{5}\right)\left(\frac{-3}{5}\right) = \boxed{\frac{-24}{25}}$

$\tan\left(\frac{a}{2}\right) = \frac{1-\cos a}{\sin a} \overset{\frac{5}{5}}{=} \frac{1-\left(\frac{-3}{5}\right)}{\frac{4}{5}} = \frac{\frac{5}{5}+\frac{3}{5}}{\frac{4}{5}} = \frac{\frac{8}{5}}{\frac{4}{5}} \cdot \frac{5}{4} = \boxed{2}$

Prove:

1. $\tan x + \cot x = \sec x \csc x$
2. $\frac{\cos 2x}{\sin^2 x} = \csc^2 x - 2$
3. $\frac{1}{2}\csc^2\frac{x}{2} = \csc^2 x + \cot x \csc x$
4. $\sec 2x = \frac{\sec^2 x}{2 - \sec^2 x}$
5. $\tan\frac{x}{2} = \frac{\tan x}{\sec x + 1}$
6. $\frac{1+\cos 2x}{\sin 2x} = \cot x$
7. $\frac{\sin x - \cos x}{\cos^2 x} = \frac{\tan^2 x - 1}{\sin x + \cos x}$
8. $\sin 3x \cos 3x = \frac{1}{2}\sin 6x$
9. $\cos^2 x - 2\sin^2 x\cos^2 x - \sin^2 x + 2\sin^4 x = \cos^2 2x$
10. $\cos 3x = 4\cos^3 x - 3\cos x$

3. $\frac{1}{2}\csc^2\left(\frac{x}{2}\right)$

$= \frac{1}{2}\left[\csc\frac{x}{2}\right]^2 = \frac{1}{2}\left[\frac{1}{\sin\frac{x}{2}}\right]^2$

$= \frac{1}{2}\left[\frac{1}{\pm\sqrt{\frac{1-\cos x}{2}}}\right]^2 = \frac{1}{2}\cdot\frac{2}{1-\cos x}$

$= \frac{1}{1-\cos x}\cdot\frac{1+\cos x}{1+\cos x} = \ldots$