Review: Prove.

$\cos 5x \cos 3x + \sin 5x \sin 3x = \cos^2 x - \sin^2 x$

$\cos(5x-3x) = \cos 2x = \cos^2 x - \sin^2 x$

LHS RHS

$\cos 3x = 4\cos^3 x - 3\cos x$

LHS $= \cos(2x+x) = \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - 2\sin x \cos x \cdot \sin x$

$= 2\cos^3 x - \cos x - 2\sin^2 x \cos x$

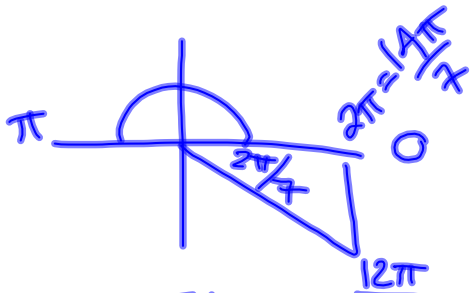
$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$ $\quad \sin^2 x + \cos^2 x = 1$, $\sin^2 x = 1 - \cos^2 x$

$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$

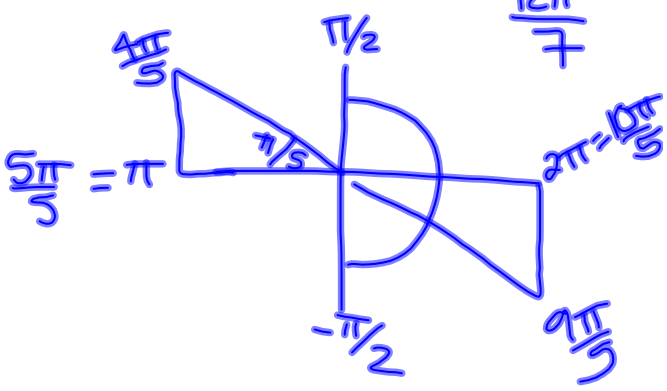
$= 4\cos^3 x - 3\cos x = RHS$

Evaluate:

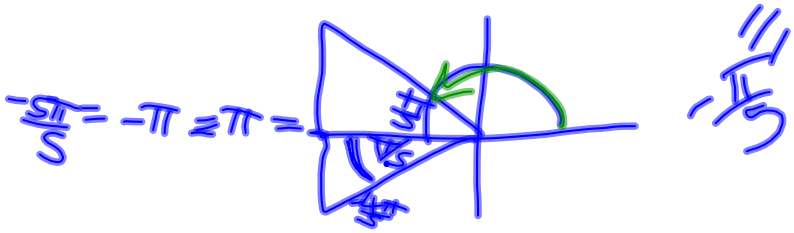
$\cos^{-1}\left(\cos\left(\frac{12\pi}{7}\right)\right) = \frac{2\pi}{7}$



$\tan^{-1}\left(\tan\left(\frac{4\pi}{5}\right)\right) = \frac{-\pi}{5}$



$\sec^{-1}\left(\sec\left(-\frac{4\pi}{5}\right)\right) = \frac{4\pi}{5}$

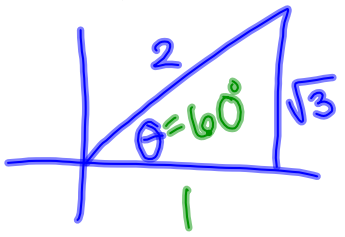


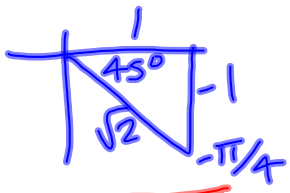
Inverse Trig Functions, cont.

1. $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right) = \boxed{\frac{1}{2}}$

θ

$= \cos 60^\circ$ //



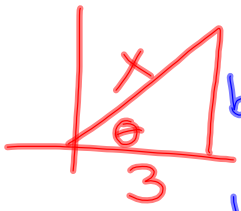


2. $\sin^{-1}\left[\tan\left(\frac{-\pi}{4}\right)\right] = \sin^{-1}(-1) = \boxed{\frac{-\pi}{2}} = -90^\circ$

3. $\tan\left(\cos^{-1}\frac{3}{x}\right)$

θ

$= \frac{\sqrt{x^2-9}}{3}$



$x > 0$

$b^2 + 3^2 = x^2$

$b^2 = x^2 - 9$

$b = \sqrt{x^2-9}$

$(x-3)(x+3)$

$(x-3)^2 = (x-3)(x-3)$

$= x^2 - 6x + 9$

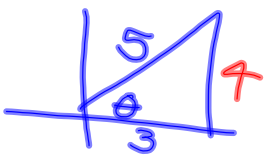
4. $\sin\left(2\cos^{-1}\frac{3}{5}\right)$

θ

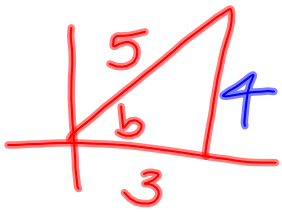
$= 2\sin\theta\cos\theta$

$= 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)$

$= \boxed{\frac{24}{25}}$



5. $\sin\left(\underbrace{\sin^{-1}\frac{1}{2}}_{a}+\underbrace{\cos^{-1}\frac{3}{5}}_{b}\right)$



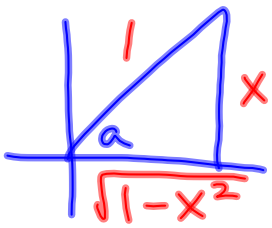
$= \sin a\cos b + \cos a\sin b$

$= \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{4}{5}\right)$

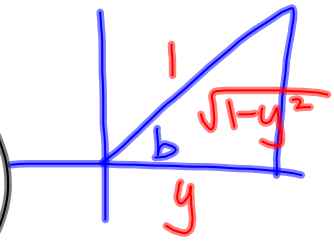
$= \frac{3}{10} + \frac{4\sqrt{3}}{10}$

$= \boxed{\frac{3+4\sqrt{3}}{10}}$

6. $\cos\left(\underbrace{\sin^{-1}\frac{x}{1}}_{a}-\underbrace{\cos^{-1}\frac{y}{1}}_{b}\right)$ $x, y > 0$



$= \cos a\cos b + \sin a\sin b$



$= \left(\frac{\sqrt{1-x^2}}{1}\right)\left(\frac{y}{1}\right) + \left(\frac{x}{1}\right)\left(\frac{\sqrt{1-y^2}}{1}\right)$

$= \boxed{y\sqrt{1-x^2} + x\sqrt{1-y^2}}$

textbook

39. $\cos^{-1}\left(\cos\left(\frac{-\pi}{4}\right)\right)$

41. $\sin^{-1}\left(\sin\frac{\pi}{5}\right) = \frac{\pi}{5}$

43. $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

45. $\sin\left(\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)\right)$ $\frac{1}{\sqrt{3}}$

47. $\tan\left(\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right)$ $\frac{1}{\sqrt{2}}$

53. $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)$

55. $\sin\left(\tan^{-1}\frac{a}{3}\right)$ $a > 0$

63. $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2} + \cos^{-1}\frac{3}{5}\right)$ $\frac{1}{\sqrt{2}}$ (a, b)

Homework:

6.5 Handout: #25-53