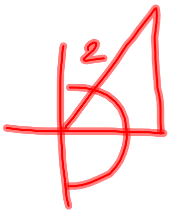
Solving Trigonometric Equations

$\sin^{-1}\left(\frac{1}{2}\right)$ versus $\sin x = \frac{1}{2}$



$= \boxed{\frac{\pi}{6}}$

Only 1 answer

$x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, -\frac{11\pi}{6}, \ldots$

infinitely many solutions!

$x = \frac{\pi}{6} + 2\pi k,\ x = \frac{5\pi}{6} + 2\pi k,$

k = an integer

6.6 Handout

Solve for $x \in [0, 2\pi)$.

2. $2\sin x = \sqrt{3}$

$\frac{2\sin x}{2} = \frac{\sqrt{3}}{2}$

$\sin x = \frac{\sqrt{3}}{2}$

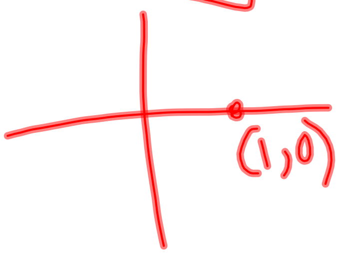
$\boxed{x = \frac{\pi}{3}, \frac{2\pi}{3}}$

4. $\cos x - 1 = 0$

$+1 \quad +1$

$\cos x = 1$

$\boxed{x = 0}$



6. $2\cancel{\sin x}\cos x = \sqrt{3}\cancel{\sin x}$

$\dfrac{2\cancel{\sin x}\cos x}{\cancel{\sin x}} = \dfrac{\sqrt{3}\cancel{\sin x}}{\cancel{\sin x}}$

$2\cos x = \sqrt{3}$

$\cos x = \dfrac{\sqrt{3}}{2}$

$\dfrac{x^2}{\cancel{x}} = \dfrac{\cancel{x}}{\cancel{x}}$

$x = 1$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, \ x = 1$

Algebra Review

$(x-2)(x-3)(x-4) = 0$

$x-2=0, \ x-3=0, \ x-4=0$

$x = 2, 3, 4$

The **Zero Product Property** states:

If $AB = 0$, then $A = 0$ or $B = 0$.

$x^2 = 9$

$x = \pm 3$

The **Square Root Theorem** states:

If $[f(x)]^2 = c$, then $f(x) = \pm\sqrt{c}$

6. $2 \sin x \cos x = \sqrt{3} \sin x$, $x \in [0, 2\pi)$

$2 \sin x \cos x - \sqrt{3} \sin x = 0$

$\sin x (2 \cos x - \sqrt{3}) = 0$

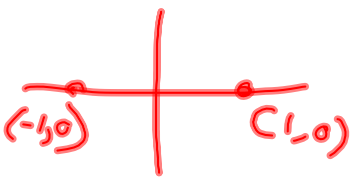
$\sin x = 0$

$\boxed{x = 0, \pi}$

$2 \cos x - \sqrt{3} = 0$

$\cos x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}, \frac{11\pi}{6}}$



8. $\cos^2 x - 1 = 0$, $x \in [0, 2\pi)$

$\cos^2 x = 1$

$\cos x = \pm 1$

$\boxed{x = 0, \pi}$

10. $\sec^2 x + \sqrt{3}\sec x - \sqrt{2}\sec x - \sqrt{6} = 0$

Let $\sec x = u$.

$\underbrace{u^2 + \sqrt{3}\,u}\ \underbrace{-\sqrt{2}\,u - \sqrt{6}} = 0$

factor by grouping

$u(u+\sqrt{3}) - \sqrt{2}(u+\sqrt{3}) = 0$

$(u+\sqrt{3})(u-\sqrt{2}) = 0$

$u+\sqrt{3} = 0 \qquad u-\sqrt{2} = 0$

$u = -\sqrt{3} \qquad u = \sqrt{2}$

$\sec x = -\sqrt{3} \qquad \sec x = \sqrt{2}$

x = some not nice ∠ in Q's 2 & 3

$\boxed{x = \frac{\pi}{4}, \frac{7\pi}{4}}$

14. $2\cos^2 x + 1 = -3\cos x$

$$2\cos^2 x + 3\cos x + 1 = 0$$

Let $u = \cos x$

$$2u^2 + 3u + 1 = 0$$

$$(2u+1)(u+1) = 0$$

$$(2\cos x + 1)(\cos x + 1) = 0$$

$$\cos x = -\frac{1}{2} \qquad \cos x = -1$$

$$\boxed{x = \frac{2\pi}{3}, \frac{4\pi}{3} \; ; \; \pi}$$

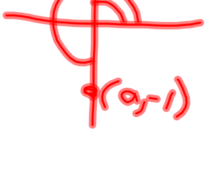
18. $4\cos^3 x = 3\cos x$

$$4\cos^3 x - 3\cos x = 0$$

$$\cos x\,(4\cos^2 x - 3) = 0$$

$$\cos x = 0 \qquad \cos^2 x = \frac{3}{4}$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}} \qquad \cos x = \pm\frac{\sqrt{3}}{2}$$



$$\boxed{x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}}$$

20. $\tan^2 x + \tan x - \sqrt{3} = \sqrt{3}\tan x$

$\tan^2 x + \tan x - \sqrt{3}\tan x - \sqrt{3} = 0$

$\tan x(\tan x + 1) - \sqrt{3}(\tan x + 1) = 0$

$(\tan x + 1)(\tan x - \sqrt{3}) = 0$

$\tan x = -1 \qquad \tan x = \sqrt{3}$

$\boxed{x = \frac{3\pi}{4}, \frac{7\pi}{4}} \qquad \boxed{x = \frac{\pi}{3}, \frac{4\pi}{3}}$

22. $\cos^4 x = \cos^2 x$

$\cos^4 x - \cos^2 x = 0$

$\cos^2 x(\cos^2 x - 1) = 0$

$\cos^2 x = 0 \qquad \cos^2 x - 1 = 0$

$\cos x = 0 \qquad \cos^2 x = 1$

$\cos x = \pm 1$

$\cos^2 x(\cos x - 1)(\cos x + 1) = 0$

$\cos^2 x = 0, \cos x = 1, \cos x = -1$

$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}, 0, \pi}$

New Directions: Find ALL the solutions (not just in $[0,2\pi)$)

62. $\sec 3x - \frac{2\sqrt{3}}{3} = 0$

$\sec(3x) = \frac{2\sqrt{3}}{3}$

$\sec(3x) = \frac{2}{\sqrt{3}}$

$3x = \frac{\pi}{6} + 2\pi k$; $3x = \frac{11\pi}{6} + 2\pi k$

$\boxed{x = \frac{\pi}{18} + \frac{2\pi k}{3} \ ; \quad x = \frac{11\pi}{18} + \frac{2\pi k}{3}}$

68. $\cos\left(2x - \frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} = \frac{-1}{\sqrt{2}}$

$2x - \frac{\pi}{4} = \frac{3\pi}{4} + 2\pi k$; $2x - \frac{\pi}{4} = \frac{5\pi}{4} + 2\pi k$

$2x = \pi + 2\pi k$ $\qquad$ $2x = \frac{3\pi}{2} + 2\pi k$

$\boxed{x = \frac{\pi}{2} + \pi k}$ $\qquad$ $\boxed{x = \frac{3\pi}{4} + \pi k}$

Homework (from 6.6 Handout)

#1-21 odd - finding solutions in $[0,2\pi)$

#61-69 odd - finding all possible solutions $(+2\pi k)$

Quiz moved to Wednesday

Test still scheduled for Friday